# INTEGRAL HESABI

Eğer $F(x)$ ve $f(x)$ gibi tanımlanmış fonksiyon varsa ve $\frac{d}{dx}F(x) = f(x)$ ise $F(x)=\int f(x)\,dx$ ifadesine $f(x)$'in belirsiz integrali denir.

$$F(x)+c=\int f(x)\,dx \quad c= \text{sbt.}$$

Eğer burada c sabitini belirleyebiliyorsak bu integral belirli bir integrale dönüşür.

Basit integrallerde temelde üç çeşit integral yapısıyla karşılaşabiliriz. Eğer integral belirli integral ise o aralıkta ve belirtme sınırlarında fonksiyonun durumu önem kazanacağından burada güçlüklerle karşılaşılabilir. Bu da seri yöntemleriyle Monte Carlo hesabı, ileye kaydı dediğimiz karmaşık sayıları kullanarak geliştirilmiş tekniklerle hesaplamaya çalışılır veya Feynman'ın yaptığı gibi bozukluk olan noktaları bazı matematiksel teknolojilerle aşılır.

1- Değişken dönüşme ile çözülebilen integraller
2- Rasyonel yani kesirli formattaki fonksiyonların integralleri
3- Kısmi integrasyon tekniğine uyabilen integraller
4- Kendine özel integraller

$$\frac{dx^n}{dx}=n x^{n-1} \Rightarrow \frac{1}{n}\cdot\frac{dx^n}{dx}= x^{n-1}$$

$$\frac{d}{dx}\left(\frac{1}{n}x^n\right)=x^{n-1} \quad \text{ise}$$

$$\boxed{\frac{1}{n}x^n+c=\int x^{n-1}dx}$$

$$\int x^3dx = \frac{1}{4}x^4+c$$

$$\boxed{\int x^k\,dx = \frac{x^{k+1}}{k+1}+c}$$

Ö $\int \sqrt[3]{x^2}\,dx = \int x^{\frac{2}{3}}\,dx = \dfrac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c = \dfrac{x^{5/3}}{5/3} + c = \dfrac{3}{5}\cdot\sqrt[3]{x^5} + c$

Ö $\dfrac{d}{dx}(\sin \omega x) = \omega \cdot \cos \omega x$

– $\dfrac{d}{dx}\left(\dfrac{1}{\omega}\cdot \sin \omega x\right) = \cos \omega x$

$$* \boxed{\int \cos \omega x\,dx = \frac{1}{\omega}\cdot \sin \omega x + c}$$

– $\dfrac{d}{dx}\left(\dfrac{1}{\omega}\cos \omega x\right) = -\sin \omega x$

$$* \boxed{\int \sin \omega x\,dx = -\frac{1}{\omega}\cos \omega x + c}$$

– $\dfrac{d}{dx}\ln x = \dfrac{1}{x} \Rightarrow * \boxed{\int \dfrac{1}{x}\,dx = \ln x + c}$

– $\dfrac{d}{dx}(e^x) = e^x + c \Rightarrow * \boxed{\int e^x dx = e^x + c}$

– $\dfrac{d}{dx}(\operatorname{arctg} x) = \dfrac{1}{1+x^2}$

$$* \boxed{\int \frac{dx}{1+x^2} = \operatorname{arctg} x + c}$$

$\dfrac{d}{dx}\left(F(x) \pm G(x)\right) = f(x) \pm g(x) \Rightarrow$

$$* \boxed{\int [(f(x) \mp g(x)]\,dx = F(x) \mp G(x) + c}$$

Değişken dönüşümüne uyabilen integraller

Ö: $\int (2x+1)^4\,dx$

$2x+1=u \qquad 2dx=du \Rightarrow dx=\frac{du}{2}$

$$\int u^4 \frac{du}{2} = \frac{1}{2}\int u^4 du = \frac{1}{2}\cdot\frac{u^5}{5}+c = \frac{(2x+1)^5}{10}+c$$

Ö: $\int \sqrt[3]{3x-2}\,dx = \int \underbrace{(3x-2)}_{u}{}^{1/3}\,dx$

$3x-2=u \qquad 3dx=du$

$dx=du/3$

$$\int u^{1/3}\frac{du}{3} = \frac{1}{3}\int u^{1/3}du = \frac{1}{3}\cdot\frac{u^{1/3+1}}{1/3+1}+c = \frac{u^{4/3}}{4}+c$$

$$=\frac{\sqrt[3]{(3x-2)^4}}{4}+c$$

<u>Sonuç:</u> $\int [f(x)]^n g(x)\,dx$

ve

$f'(x)=\alpha . g(x) \Rightarrow \alpha =$ sabit sayı

ise

$u=f(x)$ dönüşümü yapılır.

Ö: $\int (x^3+1)^6 . x^2\,dx \qquad x^3+1=u$

$3x^2dx=du \Rightarrow dx=\frac{du}{3x^2}$

$$\int u^6 . \cancel{x^2} . \frac{du}{3\cancel{x^2}} = \frac{1}{3}\int u^6 du = \frac{1}{3}\frac{(x^3+1)^7}{7}+c = \frac{(x^3+1)^7}{21}+c$$

Ö: $\int \sqrt{\sin x+1}\cdot\cos x\,dx = \int \underbrace{(\sin x+1)}_{u}{}^{1/2} . \cos x . dx$

$\sin x+1=u$
$\cos x\,dx=du$
$dx=\frac{du}{\cos x}$

$$\int u^{1/2} . \cancel{\cos x} . \frac{du}{\cancel{\cos x}} = \frac{u^{3/2}}{3/2}+c = (\sin x+1)^{3/2}.\frac{2}{3}+c$$

$$= \frac{2}{3}\sqrt[2]{(\sin x+1)^3}+c$$

Ö $\int \cos^3 x \cdot \sin x \, dx$ $\qquad \cos x = u \Rightarrow -\sin x \, dx = du$

$\int u^3 \cancel{\sin x} \cdot \left(\frac{-du}{\cancel{\sin x}}\right) = -\int u^3 du =$ $\qquad dx = \frac{-du}{\sin x}$

$-\frac{1}{4}\cos^4 x + c$

Sonuç:

$\int \left.\begin{matrix} \sin f(x) \\ \cos f(x) \end{matrix}\right\} \alpha \, f'(x) \, dx \Rightarrow \alpha = \text{sbt.}$

ise değişken dönüşümü kullanılabilir.

Ö $\int x \cdot \cos(x^2+1) \, dx$ $\qquad x^2+1 = u$

$2x\,dx = du$

$\int \cancel{x} \cdot \cos u \cdot \frac{du}{2\cancel{x}} =$ $\qquad dx = \frac{du}{2x}$

$\frac{1}{2}\int \cos u \, du = \frac{1}{2}\sin u + c = \frac{1}{2}\sin(x^2+1) + c$

Ö $\int \sin(\underbrace{x^3+x+1}_{u}) \underbrace{(3x^2+1) dx}_{du} = -\cos(x^3+x+1) + c$

$du = (3x^2+1)dx$

$\int \sin u \, du = -\cos u + c$

$= -\cos(x^3+x+1)+c$

Sonuç:

$\int g(x) e^{f(x)} dx \quad f'(x) = x \cdot g(x)$ ise dönüşüm yapılabilir.

Ö $\int x \cdot e^{x^2+1} dx =$ $\qquad x^2+1 = u$

$2x\,dx = du$

$\int x e^u \cdot \frac{du}{2x} = \frac{1}{2}\int e^u du =$ $\qquad dx = \frac{du}{2x}$

$\frac{1}{2} e^{x^2+1} + c$

Ö $\int \sin x \cdot e^{\cos x} dx =$ $\qquad$ $\cos x = u$

$\int \cancel{\sin x}\, e^u \cdot -\frac{du}{\cancel{\sin x}} =$ $\qquad$ $-\sin x\, dx = du$

$-\int e^u \cdot du = -e^{\cos x} + c$

Ö $\int \frac{e^{\operatorname{arctg} x}}{1+x^2} \cdot dx =$ $\qquad$ $\operatorname{arctg} x = u$

$\frac{1}{1+x^2} dx = du$

$\int \frac{1}{\cancel{1+x^2}} \cdot e^u \cdot du \cancel{(1+x^2)} =$ $\qquad$ $dx = du(1+x^2)$

$\int e^u \cdot du = e^u + c = e^{\operatorname{arctg} x} + c$

Rasyonel İntegrallerin Hesabı

$\int \frac{f(x)}{g(x)} dx$ $\qquad$ $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots$

$g(x) = b_m x^m + b_{m-1} x^{m-1} + b_{m-2} x^{m-2} + \dots$

1) Eğer $n \geq m$ $\qquad$ $\begin{array}{r|l} f(x) & g(x) \\ \hline & k(x) \\ m(x) & \end{array}$

$\frac{f(x)}{g(x)} = k(x) + \frac{m(x)}{g(x)}$ $\qquad$ $\int \frac{f(x)}{g(x)} dx = \int k(x) dx + \int \frac{m(x)}{g(x)} dx$

2) Eğer $n < m$ ise

i) eğer fark 1 ise paydanın türevi paya benzetilmeye çalışılır

ii) eğer fark 1'den büyük ise kesir basit kesirlerine ayrılır.

Ö $\int \frac{x^3}{x+1} dx =$ $\qquad$ $\frac{x^3}{x+1} = x^2 - x + 1 - \frac{1}{x+1}$

$\begin{array}{r|l} x^3 & x+1 \\ \hline x^3 + x^2 & x^2 - x + 1 \\ -x^2 & \\ -x^2 - x & \\ +x & \\ +x + 1 & \end{array}$

$\int \frac{x^3}{x+1} dx = \frac{x^3}{3} - \frac{x^2}{2} + x - \int \frac{1}{x+1} dx + c$

$= \frac{x^3}{3} - \frac{x^2}{2} + x - \ln(x+1) + c$

Ö. $\int \frac{x^2}{x^2+1}\,dx$

$$= \int \frac{x^2+1-1}{x^2+1}\,dx = \int \left(1 - \frac{1}{x^2+1}\right) dx = x - \int \frac{1}{1+x^2}\,dx + c = x - \operatorname{arctg} x + c$$

Ö $\int \frac{x}{x^2+4}\,dx =$

$$\frac{1}{2}\int \frac{2x}{x^2+4}\,dx = \frac{1}{2}\ln(x^2+4) + \ln c$$

→ sabitin yerine $\ln c$ de yazılabilir

$$= \ln(x^2+4)^{\frac{1}{2}} + \ln c$$

$$= \ln c\sqrt{x^2+4}$$

Ö $\int \frac{x^2+1}{x^2+4}\,dx = \int \frac{x^2+1+3-3}{x^2+4}\,dx = \int \left(1 - \frac{3}{x^2+4}\right) dx = x - 3\int \frac{1}{x^2+4}\,dx + c$

$$= \int \frac{1}{x^2+4}\,dx = \int \frac{1}{4\left(\frac{x^2}{4}+1\right)} = \frac{1}{4}\int \frac{dx}{\frac{x^2}{4}+1}$$

$$\left(\frac{x}{2}\right) = t \Rightarrow \quad \frac{1}{2}\,dx = dt$$

$$dx = 2dt$$

$$= \frac{1}{4}\int \frac{2dt}{t^2+1} = \frac{1}{2}\int \frac{dt}{t^2+1} = \frac{1}{2}\operatorname{arctg} t = \frac{1}{2}\operatorname{arctg}\frac{x}{2}$$

$$= \int \frac{x^2+1}{x^2+4} = x - \frac{3}{2}\operatorname{arctg}\frac{x}{2} + c$$

Ö $\int \frac{dx}{x^2+x+3}$ $\qquad x^2+x+3 = x^2+x+\frac{1}{4} - \frac{1}{4} + 3 = \left(x+\frac{1}{2}\right)^2 + \frac{11}{4}$

$$\int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{11}{4}} = \int \frac{du}{u^2+\frac{11}{4}} = \int \frac{du}{\frac{11}{4}\left(u^2\frac{4}{11}+1\right)} = \frac{4}{11}\int \frac{du}{\frac{4}{11}u^2+1} =$$

$x+\frac{1}{2} = u$

$dx = du$

$$\sqrt{\frac{4}{11}}\,u = t \Rightarrow \sqrt{\frac{4}{11}}\,du = dt \Rightarrow du = \sqrt{\frac{11}{4}}\,dt$$

$$= \frac{4}{11}\int \frac{\sqrt{\frac{11}{4}}\,dt}{t^2+1} = \sqrt{\frac{4}{11}}\int \frac{dt}{1+t^2} = \sqrt{\frac{4}{11}}\operatorname{arctg} t + c$$

$$= \sqrt{\frac{4}{11}}\operatorname{arctg}\sqrt{\frac{4}{11}}\,u + c = \sqrt{\frac{4}{11}}\operatorname{arctg}\left[\sqrt{\frac{4}{11}}\left(x+\frac{1}{2}\right)\right] + c$$

$$\int \frac{x+2}{x^2+x+3}\,dx = \frac{1}{2}\int \frac{2x+4}{x^2+x+3}\,dx = \frac{1}{2}\int \frac{2x+1+3}{x^2+x+3}\,dx =$$

$$= \frac{1}{2}\int \frac{2x+1}{x^2+x+3}\,dx + \frac{3}{2}\int \frac{1}{x^2+x+3}\,dx =$$

$$= \frac{1}{2}\ln(x^2+x+3) + \frac{3}{2}\sqrt{\frac{4}{11}}\,\mathrm{arctg}\left[\sqrt{\frac{4}{11}}\left(x+\frac{1}{2}\right)\right] + c$$

$$\int \frac{1}{x^2-x-2}\,dx = \int \frac{1}{(x-2)(x+1)}\,dx =$$

$$\frac{1}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x+1)} = \frac{Ax+A+Bx-2B}{(x-2)(x+1)} = \frac{(A+B)x+A-2B}{(x-2)(x+1)}$$

$$A+B=0 \Rightarrow B=-\frac{1}{3}$$

$$A-2B=1 \Rightarrow A=\frac{1}{3}$$

$$= \frac{1}{3}\left[\int \frac{1}{x-2}\,dx - \int \frac{1}{x+1}\,dx\right] = \frac{1}{3}\left[\ln(x-2) - \ln(x+1)\right] + \ln c$$

$$= \ln\left(\frac{x-2}{x+1}\right)^{1/3} + \ln c = \ln c.\left(\frac{x-2}{x+1}\right)^{1/3}$$

$$\int \frac{x+3}{x^2-x-2}\,dx = \int \frac{x+3}{(x-2)(x+1)}\,dx$$

$$\frac{x+3}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x+1)} = \frac{x(A+B)+A-2B}{(x-2)(x+1)}$$

$$\left.\begin{array}{l} A+B=1 \\ A-2B=3 \end{array}\right\rangle \begin{array}{l} B=-\frac{2}{3} \\ A=\frac{5}{3} \end{array}$$

$$= \frac{5}{3}\ln(x-2) - \frac{2}{3}\ln(x+1) + c$$

$$\int \frac{x+2}{x(x^2-1)}\,dx =$$

$$\frac{x+2}{x(x^2-1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} \quad ..$$

$$\int \frac{x^2+1}{x(x^2-1)}\,dx = \ln\left(\frac{x^2-1}{x}\right)+C$$

$$\int \frac{x}{\underbrace{(x^2+2x-3)}_{(x+3)(x-1)}(x+5)} = \frac{1}{18}\ln\frac{(x-1)^2(x+3)^2}{(x+5)^2}$$

$$\int \frac{x+2}{x^3+x^2+4x+4}\,dx =$$

| | | | | |
|---|---|---|---|---|
| $-1$ | 1 | 1 | 4 | 4 |
| | | $-1$ | 0 | $-4$ |
| | 1 | 0 | 4 | 0 |

$$\Rightarrow \int \frac{x+2}{(x+1)(x^2+4)}\,dx$$

$$\frac{x^2+1}{x(x^2-1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$= \frac{Ax^2-A+Bx^2+Bx+Cx^2-Cx}{x(x^2-1)}$$

$$A=-1,\quad B=C=1$$

$$\int\frac{x^2+1}{x(x^2-1)} = \int\left(-\frac{1}{x}+\frac{1}{x-1}+\frac{1}{x+1}\right)dx$$

$$= -\ln x+\ln(x-1)+\ln(x+1)$$

$$= \ln\left(\frac{x^2-1}{x}\right)+C$$

$$\frac{x+2}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$$

$$\frac{Ax^2+4A+Bx^2+Cx+Bx+C}{(x+1)(x^2+4)} = \frac{x^2(A+B)+(B+C)x+4A+C}{(x+1)(x^2+4)}$$

$A+B=0$
$B+C=1$
$4A+C=2$

$$\begin{array}{r} 4A-B=1 \\ A+B=0 \\ \hline 5A=1 \end{array}$$

$$A=\frac{1}{5}\qquad B=-\frac{1}{5}\qquad C=\frac{6}{5}$$

$$\frac{x+2}{(x+1)(x^2+4)} = \frac{1/5}{x+1} + \frac{1}{5}\,\frac{-x+6}{x^2+4}$$

$$= \frac{1}{5}\ln(x+1)+\left[-\frac{1}{5}\int\frac{x-6}{x^2+4}\,dx\right] = \int\frac{x-6}{x^2+4}\,dx = \frac{1}{2}\int\frac{2x-12}{x^2+4}\,dx$$

$$= \frac{1}{2}\int\frac{2x}{x^2+4}\,dx - \frac{12}{2}\int\frac{1}{x^2+4}\,dx = \frac{1}{2}\ln(x^2+4) - \frac{12}{2}$$

$$\int\frac{dx}{x^2+4} = \frac{1}{4}\int\frac{dx}{\frac{x^2}{4}+1}$$

$$\frac{x}{2}=t\qquad \frac{x^2}{4}=t^2\qquad dx=2dt$$

$$= \frac{1}{4}\int\frac{2dt}{t^2+1} = \frac{1}{2}\operatorname{arctg} t = \frac{1}{2}\operatorname{arctg}\frac{x}{2}$$

$$= \frac{1}{5}\ln(x+1) - \frac{1}{5}\left[\frac{1}{2}\ln(x^2+4) - 3\operatorname{arctg}\frac{x}{2}\right]$$

Ö $\displaystyle\int \frac{x}{x^3+x-2}\,dx$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & +2 \\ \hline & 1 & 1 & 2 & 0 \end{array} \qquad \Rightarrow \int \frac{x}{(x-1)(x^2+x+2)}\,dx$$

$$= \frac{A}{(x-1)} + \frac{Bx+C}{x^2+x+2}$$

$$* \ \frac{x-1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$

$$* \ \frac{x}{(x+1)^2(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)}$$

$$* \ \frac{x+3}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$$

$$* \ \frac{x-2}{(x-1)^2(x^2+2x+5)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+2x+5}$$

*Trigonometrik Fonksiyonların İntegrali*

1. $\displaystyle\int \sin^m x\,dx$ $\qquad m =$ tek ise

$\displaystyle\int \cos^m x\,dx$ $\qquad m =$ tek ise

Ö $\displaystyle\int \sin^3 x\,dx = \int \sin^2 x \cdot \sin x\,dx = \int (1-\cos^2 x)\sin x\,dx$

$$= \int \sin x\,dx - \int \cos^2 x \sin x\,dx \Rightarrow \quad \begin{array}{l} \cos x = u \\ -\sin x\,dx = du \\ dx = -\dfrac{du}{\sin x} \end{array}$$

$$= \cos x + \int u^2 \cdot \cancel{\sin x} \cdot \frac{du}{\cancel{\sin x}} = \cos x + \int u^2\,du$$

$$= \cos x + \frac{\cos^3 x}{3} + C$$

Ö $\int \cos^5 x\,dx = \int \cos^4 x \cdot \cos x\,dx = \int (1-\sin^2 x)^2 \cos x\,dx$

$= \int (1 - 2\sin^2 x + \sin^4 x)\cos x\,dx \ \ldots$

(2) $\left.\begin{array}{l} \int \sin^n x\,dx \\ \int \cos^n x\,dx \end{array}\right\}$ n = çift ise bu durumda yarım açı formülüne gidilir ve sonradan çıkacak olan tek kuvvetler bir önceki durumdaki gibi çözülür.

Ö $\int \cos^2 x\,dx$

$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1 \Rightarrow \boxed{\cos^2 x = \frac{1+\cos 2x}{2}}$

$= \frac{1}{2}\int (1+\cos 2x)\,dx = \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + c$

Ö $\int \sin^4 x\,dx \qquad \cos 2x = 1 - 2\sin^2 x$

$= \frac{1}{4}\int (1-\cos 2x)^2 dx \qquad \boxed{\sin^2 x = \frac{1-\cos^2 x}{2}}$

$= \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\,dx$

$= \frac{1}{4}(x - \sin 2x) + \frac{1}{4}\int \cos^2 2x\,dx =$

$= \frac{1}{4}(x - \sin 2x) + \int \frac{1+\cos 4x}{2}\,dx = \frac{1}{4}(x - \sin 2x) + \frac{1}{8}\left(x + \frac{1}{4}\sin 4x\right) + c$

Ö $\int \sin^4 x \cos x\,dx = \int u^4 \cos x \frac{du}{\cos x} = \int u^4 du = \frac{\sin^5 x}{5} + c$

$\sin x = u$
$\cos x\,dx = du$

Ö $\int \frac{\sin x}{1+\cos x}\,dx = \int \frac{\sin x}{u} \cdot \frac{du}{-\sin x} = -\ln(1+\cos x) + c$

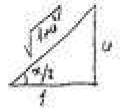
$1+\cos x = u$
$-\sin x\,dx = du$

Ö $\int \frac{dx}{\sqrt{4-x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}} = \frac{1}{\cancel{2}}\int \frac{\cancel{2}du}{\sqrt{1-u^2}} = \arcsin\left(\frac{x}{2}\right) + c$

$\frac{x}{2} = u$

$dx = 2du$

Ö $\int \frac{dx}{\sin x} =$




$u = tg\frac{x}{2}$

$du = \frac{1}{2}\left(1+tg^2\frac{x}{2}\right)dx$

$du = \frac{1}{2}(1+u^2)dx$

$dx = \frac{2du}{1+u^2}$

$\sin x = 2\sin\frac{x}{2} \cdot \cos\frac{x}{2}$

$\sin x = 2 \cdot \frac{u}{\sqrt{1+u^2}} \cdot \frac{1}{\sqrt{1+u^2}}$

$\sin x = \frac{2u}{1+u^2}$

$\int \frac{dx}{\sin x} = \frac{\cancel{2}du}{\cancel{1+u^2}} \cdot \frac{\cancel{1+u^2}}{\cancel{2}u} = \ln\left[tg\left(\frac{x}{2}\right)\right] + c$

Ö $\int \frac{dx}{\cos 2x} = \frac{1}{2}\int \frac{dt}{\cos t}$

$2x = t$

$2dx = dt$

$u = tg\frac{t}{2}$

$dt = \frac{2du}{1+u^2}$

$\cos t = 2\cos^2\frac{t}{2} - 1 = 2\left(\frac{1}{\sqrt{1+u^2}}\right)^2 - 1$

$\cos t = \frac{2}{1+u^2} - 1 = \frac{1-u^2}{1+u^2}$

$= \frac{1}{2}\int \frac{dt}{\cos t} = \int \frac{2du}{\cancel{1+u^2}} \cdot \frac{\cancel{1+u^2}}{1-u^2} = \int \frac{2du}{1-u^2} = 2\int \frac{du}{1-u^2}$

$= \frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u} = \frac{A + Au + B - Bu}{(1-u)(1+u)} = \frac{u(A-B) + A+B}{1-u^2}$

$A-B=0 \quad A+B=1$

$A=B=1$

$= \cancel{2}\,\frac{1}{\cancel{2}}\int\left[\frac{1}{1-u} + \frac{1}{1+u}\right]du = -\ln(1-u) + \ln(1+u) + c$

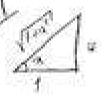
$= \ln\frac{(1+u)}{1-u} + c$

Devamı Yanda

$$\int \frac{dt}{\cos t} = \ln \frac{\left(1+\operatorname{tg}\frac{t}{2}\right)}{\left(1-\operatorname{tg}\frac{t}{2}\right)} + c$$

$$\int \frac{dx}{\cos 2x} = \frac{1}{2}\int \frac{dt}{\cos t} = \frac{1}{2}\ln\left(\frac{1+\operatorname{tg}x}{1-\operatorname{tg}x}\right) + c$$

2. yol




$$\int \frac{dx}{\cos 2x} =$$

$u = \operatorname{tg}x$ $\qquad$ $\cos 2x = 2\cos^2 x - 1$

$du = (1+\operatorname{tg}^2 x)dx$ $\qquad$ $\cos 2x = 2\left(\frac{1}{\sqrt{1+u^2}}\right)^2 - 1 = \frac{2}{1+u^2} - 1 = \frac{1-u^2}{1+u^2}$

$du = (1+u^2)dx$

$dx = \frac{du}{1+u^2}$

$$\int \frac{dx}{\cos 2x} = \int \frac{du}{\cancel{1+u^2}} \cdot \frac{\cancel{1+u^2}}{1-u^2} = \frac{1}{2}\ln\left(\frac{1+u}{1-u}\right) + c = \frac{1}{2}\ln\left(\frac{1+\operatorname{tg}x}{1-\operatorname{tg}x}\right) + c$$

$$\int \frac{dx}{\sin x + \cos x} =$$

$u = \operatorname{tg}\frac{x}{2}$ $\qquad$ $\sin x = \frac{2u}{1+u^2}$ $\qquad$ $\cos x = \frac{1-u^2}{1+u^2}$

$dx = \frac{2du}{1+u^2}$

$$\int \frac{dx}{\sin x + \cos x} = \frac{2dx/1+u^2}{\frac{2u}{1+u^2} + \frac{1-u^2}{1+u^2}} = 2\int \frac{du}{2u+1-u^2} = 2\int \frac{du}{2-(1-u)^2} =$$

$u - 1 = t$ $\qquad$ $= 2\int \frac{dt}{2-t^2} = 2\int \frac{dt}{2\left(1-\left(\frac{t}{\sqrt{2}}\right)^2\right)} = \int \frac{dt}{1-\left(\frac{t}{\sqrt{2}}\right)^2}$

$du = dt$

$\frac{t}{\sqrt{2}} = z$ $\qquad$ $\sqrt{2}\int \frac{dz}{1-z^2} = \frac{\sqrt{2}}{2}\ln\left(\frac{1+z}{1-z}\right) + c$

$dt = \sqrt{2}\,dz$

$$= \frac{\sqrt{2}}{2}\cdot\ln\left(\frac{1+\frac{t}{\sqrt{2}}}{1-\frac{t}{\sqrt{2}}}\right) + c = \frac{\sqrt{2}}{2}\ln\left(\frac{1+\frac{u-1}{\sqrt{2}}}{1-\frac{u-1}{\sqrt{2}}}\right) + c = \frac{\sqrt{2}}{2}\ln\left(\frac{1+\frac{\operatorname{tg}\frac{x}{2}-1}{\sqrt{2}}}{1+\frac{\operatorname{tg}\frac{x}{2}-1}{\sqrt{2}}}\right) + c$$

Ö $\displaystyle\int \frac{\sin x}{1+\sin x}\,dx = \int \frac{2u/1+u^2}{1+(2u/1+u^2)} \cdot \frac{2\,du}{1+u^2} = \int \frac{2u}{(1+u)^2} \cdot \frac{2\,du}{(1+u^2)}$

$u = tg\frac{x}{2}$ $\displaystyle = 4\int \frac{u\,du}{(1+u)^2(1+u^2)} =$

$$\frac{u}{(1+u)^2(1+u^2)} = \frac{A}{\underset{(1+u)(1+u^2)}{(1+u)}} + \frac{B}{(1+u)^2} + \frac{Cu+D}{1+u^2}$$

$$= \frac{A(1+u)(1+u^2) + B(1+u^2) + Cu + D(1+u)^2}{P^2} = \dots$$

Ö $\displaystyle\int \frac{\sin x}{1+2\sin x}\,dx = \frac{1}{2}\int \frac{2\sin x}{1+2\sin x}\,dx = \frac{1}{2}\int \frac{2\sin x+1-1}{1+2\sin x}\,dx = \frac{1}{2}\int\left[1 - \frac{1}{(1+2\sin x)}\right]dx$

$$= \frac{1}{2}x - \frac{1}{2}\int \frac{1}{1+\sin 2x}\,dx =$$

$$\int \frac{1}{1+2\sin x}\,dx = \int \frac{1}{1+2\frac{2u}{1+u^2}} \cdot \frac{2\,du}{1+u^2} = 2\int \frac{du}{\frac{1+u^2+4u}{1+u^2}} = 2\int \frac{du}{1+4u+u^2}$$

$u = tg\frac{x}{2}$

$$= 2\int \frac{du}{(u+2)^2 - 3} = 2\int \frac{dt}{t^2 - 3} = \frac{2}{3}\int \frac{dt}{\left(\frac{t}{\sqrt{3}}\right)^2 - 1} =$$

$u+2 = t$

$du = dt$ $\displaystyle = \frac{2}{\sqrt{3}}\int \frac{dz}{z^2-1} = -\frac{2}{\sqrt{3}}\int \frac{dz}{1-z^2} = -\frac{1}{\sqrt{3}}\ln\left(\frac{1+z}{1-z}\right) + c$

$\frac{t}{\sqrt{3}} = z$

$dt = \sqrt{3}\,dz$ $\displaystyle = -\frac{1}{\sqrt{3}}\ln\left(\frac{1+\frac{t}{\sqrt{3}}}{1-\frac{t}{\sqrt{3}}}\right) + c = -\frac{1}{\sqrt{3}}\ln\left(\frac{1+\frac{u+2}{\sqrt{3}}}{1-\frac{u+2}{\sqrt{3}}}\right) + c$

$$= -\frac{1}{\sqrt{3}}\ln\left(\frac{1+\frac{tg\frac{x}{2}+2}{\sqrt{3}}}{1-\frac{tg\frac{x}{2}+2}{\sqrt{3}}}\right) + c$$

$$= \int \frac{\sin x}{1+2\sin x} = \frac{1}{2}x - \frac{1}{2\sqrt{3}}\ln\left(\frac{1+\frac{tg\frac{x}{2}+2}{\sqrt{3}}}{1-\frac{tg\frac{x}{2}+2}{\sqrt{3}}}\right) + c$$

Ö $\displaystyle\int \frac{\cos x}{4-3\cos x}\,dx = -\frac{1}{3}\int \frac{-3\cos x}{4-3\cos x}\,dx = -\frac{1}{3}\int \frac{4-3\cos x-4}{4-3\cos x}\,dx$

$\displaystyle = -\frac{1}{3}\int\left(1-\frac{4}{4-3\cos x}\right)dx = -\frac{1}{3}x + \frac{4}{3}\int\frac{dx}{4-3\cos x}$

$\displaystyle\int\frac{dx}{4-3\cos x} = \int\frac{2du/(1+u^2)}{4-3\frac{1-u^2}{1+u^2}} = 2\int\frac{du}{1+7u^2} = 2\int\frac{du}{1+(\sqrt{7}u)^2}$

$u = tg\,\frac{x}{2}$ $\displaystyle = \frac{2}{\sqrt{7}}\int\frac{dt}{1+t^2} = \frac{2}{\sqrt{7}}\,arctg\,t = \frac{2}{\sqrt{7}}\,arctg\left(\sqrt{7}\,tg\,\frac{x}{2}\right)$

$\sqrt{7}u = t$

$\sqrt{7}du = dt$

$\displaystyle\int\frac{\cos x}{4-3\cos x} = -\frac{1}{3}x + \frac{4}{3}\cdot\frac{2}{\sqrt{7}}\cdot arctg\left(\sqrt{7}\,tg\,\frac{x}{2}\right)$

$\displaystyle = -\frac{1}{3}x + \frac{8}{3\sqrt{7}}\,arctg\left(\sqrt{7}\,tg\,\frac{x}{2}\right)$

KISMİ İNTEGRASYON

$d(uv) = udv + du\,v$

$udv = d(uv) - v\,du$

$\int udv = \int d(uv) - \int v\,du$

$\boxed{\int udv = uv - \int v\,du}$

Ö $\int x.e^x\,dx =$ $\Rightarrow$ $u = x$ $\quad dv = e^x dx$

$du = dx$ $\quad v = \int e^x dx = e^x$

1.yol

$\int x.e^x dx = x.e^x - \int e^x dx = x.e^x - e^x = e^x(x-1) + c$

2yol

$e^x = u$ $\quad x\,dx = dv$

$du = e^x dx$ $\quad v = \int x\,dx = \frac{x^2}{2}$

$\displaystyle\int x e^x dx = e^x\frac{x^2}{2} - \int\frac{x^2}{2}\cdot e^x dx = \frac{x^2}{2}e^x - \frac{1}{2}\int x^2\cdot e^x dx$

bu yöntem daha zor.
Seçim çok önemli!

Ö $\int \ln x \, dx =$ $\qquad u = \ln x \qquad dv = dx$

$\qquad du = \frac{1}{x} dx \qquad v = \int dx = x$

1.yol $\int \ln x \, dx = \ln x \cdot x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} dx + c = x \ln x - x + c = x(\ln x - 1) + c$

doğru $(x \ln x - x)' = \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}} - 1 = \ln x$

Ç $\int \operatorname{arctg} x \, dx \Rightarrow$ $\quad u = \operatorname{arctg} x \qquad dv = dx$

$\qquad du = \frac{1}{1+x^2} dx \qquad v = x$

$\int \operatorname{arctg} x \, dx = x \cdot \operatorname{arctg} x - \int x \frac{1}{1+x^2} dx + c = x \operatorname{arctg} x - ?$

$\frac{1}{2} \int \frac{2x}{1+x^2} dx = \frac{1}{2} \cdot \ln(1+x^2) + c$

$\int \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + c$

Ö $\int x \cdot \ln x \, dx =$ $\qquad u = \ln x \qquad dv = x \, dx$

$\qquad du = \frac{1}{x} dx \qquad v = \frac{x^2}{2}$

$\int x \cdot \ln x \, dx = \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$

Ö $\int x \cdot \sin 2x \, dx = -\frac{x}{2} \cos 2x - \int -\frac{1}{2} \cos 2x \, dx + c$ $\qquad x = u \quad du = dx \qquad dv = \sin 2x \, dx \quad v = -\frac{1}{2} \cos 2x$

$= -\frac{x}{2} \cos 2x + \frac{1}{2} \int \cos 2x \, dx + c$

$= -\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x + c$

Ö $\int x^2 \operatorname{arctg} x \, dx =$ $\qquad u = \operatorname{arctg} x \qquad dv = x^2 dx$

$\qquad du = \frac{1}{1+x^2} dx \qquad v = \frac{x^3}{3}$

$$\int x^2 \cdot \operatorname{arctg} x\, dx = \frac{x^3}{3} \operatorname{arctg} x - \frac{1}{3}\int x^3 \cdot \frac{1}{1+x^2}\, dx$$

$$\int \frac{x^3}{1+x^2}\, dx \qquad \begin{array}{r|l} x^3 & 1+x^2 \\ \cline{2-2} x^3+x & x \\ \hline -x & \end{array} \qquad \frac{x^3}{1+x^2} = x - \frac{x}{1+x^2}$$

$$\int \left(x - \frac{x}{1+x^2}\right) dx = \frac{x^2}{2} - \frac{1}{2}\ln(1+x^2) + c$$

$$\int x^2 \operatorname{arctg} x\, dx = \frac{x^3}{3} \operatorname{arctg} x - \frac{1}{3}\left[\frac{x^2}{2} - \frac{1}{2}\ln(1+x^2)\right] + c$$

Ö $\int x^2 \cdot e^x dx =$

$x^2 = u \qquad e^x dx = dV$

$du = 2x\,dx \qquad V = e^x$

$$\int x^2 e^x dx = x^2 e^x - \int e^x\, 2x\,dx = x^2 e^x - 2\underbrace{\int x e^x dx}_{?}$$

$\int x e^x dx \Rightarrow \begin{matrix} x = u \\ dx = du \end{matrix} \qquad \begin{matrix} e^x dx = dV \\ V = e^x \end{matrix}$

$\int x e^x = x \cdot e^x - e^x$

$$\int x^2 e^x dx = x^2 e^x - 2 \cdot x e^x - 2e^x = e^x(x^2 - 2x - 2) + c$$

Ö $\int x^2 \cos 3x\, dx \qquad \begin{matrix} x^2 = u \\ du = 2x\,dx \end{matrix} \qquad \begin{matrix} dV = \cos 3x\, dx \\ V = \frac{1}{3}\sin 3x \end{matrix}$

$$\int x^2 \cos 3x\, dx = \frac{x^2}{3} \sin 3x - \frac{2}{3}\underbrace{\int x \sin 3x\, dx}_{?}$$

$\int x \sin 3x\, dx = \qquad \begin{matrix} x = u \\ du = dx \end{matrix} \qquad \begin{matrix} dV = \sin 3x\, dx \\ V = -\frac{1}{3}\cos 3x \end{matrix}$

$\int x \sin 3x\, dx = -\frac{x}{3}\cos 3x + \frac{1}{3}\int \cos 3x\, dx + c$

$= -\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x$

$$\int x^2\cos 3x\,dx = \frac{x^2}{3}\sin 3x - \frac{2}{3}\left[-\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x\right] + C$$

Ö $\int e^{2x}.\sin x\,dx$ $\qquad u = e^{2x} \qquad dv = \sin x\,dx$

$\qquad du = 2e^{2x}dx \qquad v = -\cos x$

$$= -e^{2x}.\cos x + 2.\int \underbrace{e^{2x}.\cos x\,dx}_{?}$$

$\int e^{2x}.\cos x\,dx =$ $\qquad e^{2x} = u \qquad dv = \cos x\,dx$

$\qquad du = 2e^{2x}dx \qquad v = \sin x$

$$\int e^{2x}\cos x\,dx = e^{2x}.\sin x - 2\int \sin x.e^{2x}dx$$

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2.\left[e^{2x}\sin x - 2\int e^{2x}\sin x\,dx\right] + C$$

$$5\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x + C$$

$$\int e^{2x}\sin x\,dx = \frac{-e^{2x}}{5}\cos x + \frac{2e^{2x}}{5}\sin x + C$$

Ö $\int e^{3x}\cos 2x\,dx =$ $\qquad e^{3x} = u \qquad \cos 2x\,dx = dv$

$\qquad du = 3.e^{3x}dx \qquad v = \frac{\sin 2x}{2}$

$$\int e^{3x}.\cos 2x\,dx = e^{3x}.\frac{\sin 2x}{2} - \frac{3}{2}\int \underbrace{\sin 2x.e^{3x}dx}_{?}$$

$\int \sin 2x\, e^{3x}dx$ $\qquad e^{3x} = u \qquad \sin 2x\,dx = dv$

$\qquad du = 3.e^{3x}dx \qquad v = -\frac{\cos 2x}{2}$

$$\int \sin 2x\, e^{3x}dx = -e^{3x}.\frac{\cos 2x}{2} + 3\int \frac{\cos 2x}{2}.e^{3x}dx$$

$$\int e^{3x}\cos 2x\,dx = e^{3x}.\frac{\sin 2x}{2} - \frac{3}{2}\left[-e^{3x}.\frac{\cos 2x}{2} + \frac{3}{2}\int \frac{\cos 2x}{} . e^{3x}dx\right]$$

$$\int e^{3x}\cos 2x\,dx = \frac{1}{2}e^{3x}\sin 2x + \frac{3}{4}e^{3x}\cos 2x - \frac{9}{4}\int \cos 2x.e^{3x}dx$$

$$\frac{13}{4}\int e^{3x}\cos 2x\,dx = \frac{2}{13}e^{3x}\sin 2x + \frac{3}{13}e^{3x}\cos 2x + C$$

Ö: $\int \arcsin 2x\,dx =$ $\quad \arcsin 2x = u \quad d\vartheta = dx$

$du = \dfrac{2}{\sqrt{1+4x^2}}dx \quad \vartheta = x$

$$\int \arcsin 2x\,dx = x.\arcsin 2x - 2\underbrace{\int \frac{x}{\sqrt{1-4x^2}}\,dx}_{?}$$

$$\int x.(1-4x^2)^{-1/2}\,dx = \qquad 1-4x^2 = t$$

$$\int x.\,t^{-1/2}\frac{dt}{-8x} = -\frac{1}{8}\frac{t^{1/2}}{1/2} + c = -\frac{1}{4}(1-4x^2)^{1/2} + c \qquad dx(-8x) = dt$$

$$\int \arcsin 2x\,dx = x\arcsin 2x + \frac{1}{2}(1-4x^2)^{1/2} + c$$

Ö: $y = e^x$ fonksiyonunun grafiğini çizelim.

1- $(-\infty, +\infty)$

2- $\lim_{x\to-\infty} y = e^{-\infty} = 0$

$\lim_{x\to+\infty} y = e^{\infty} = \infty$

Eğer $x\to-\infty$ ve $x\to+\infty$ için $y$ belirli bir değere gitmiş olsaydı burada düşey asimptot olacaktı.
Eğer $x\to\infty$ ve $x\to-\infty$'a giderken $y\to\infty$ ve $-\infty$'a gitseydi ~~burada eğik asimptot ihtimali olacaktı. Ancak burada~~ da bu ihtimali gözden geçirmemiz gerekir.

$\lim_{x\to\infty} m = \lim_{x\to\infty} \frac{y}{x} = \lim_{x\to\infty} \frac{e^x}{x} = \frac{\infty}{\infty} \Rightarrow \lim_{x\to\infty} e^x = \infty$

3- $y' = e^x > 0$ Kök yok ama fonksiyon artandır. eğik asimptot yok

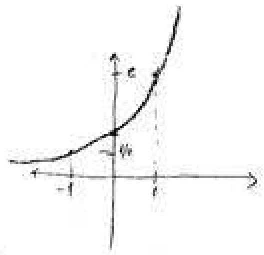
4- $y'' = e^x > 0$ $\cup$ Konveks

5- $x = 0 \Rightarrow y = 1$

$x = 1 \Rightarrow y = e$

$x = -1 \Rightarrow y = \frac{1}{e}$

| $x$ | $-\infty$ | $-1$ | $0$ | $1$ | $+\infty$ |
|---|---|---|---|---|---|
| $y'$ | | | + | | |
| $y''$ | | | + | | |
| $y$ | $0$ ↗ | $1/e$ ↗ | $1$ ↗ | $e$ ↗ | $\infty$ |




Ödev $y=e^{-x}$ 'in grafiğini çiziniz.

Ö $y=e^{-\frac{1}{x-1}}$

1- $(-\infty,1)\cup(1,+\infty)$ çünkü 1 paydayı sıfır yapar.

2. $\lim\limits_{x\to-\infty} e^{0}=1$

$\lim\limits_{x\to+\infty} e^{0}=1$ } yatay asimptot

$\lim\limits_{x\to1^{+}} y=\lim\limits_{\varepsilon\to0} e^{-\frac{1}{(1+\varepsilon-1)}}=\lim\limits_{\varepsilon\to0} e^{-\frac{1}{\varepsilon}}=e^{-\infty}=0$

$\lim\limits_{x\to1^{-}} y=\lim\limits_{\varepsilon\to0} e^{-\frac{1}{(1-\varepsilon-1)}}=\lim\limits_{\varepsilon\to0} e^{\frac{1}{\varepsilon}}=e^{\infty}=\infty$

} Sağdan ve soldan limit sonsuzluğa gitseydi düşey asimptot olurdu.

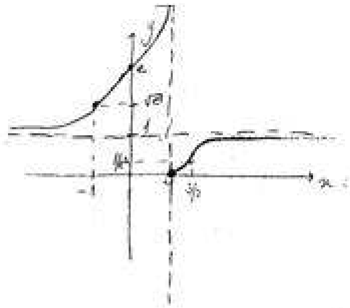
3- $y'=\frac{1}{(x-1)^2}\cdot e^{-\frac{1}{x-1}}\neq0\Rightarrow>0$ artan fonksiyon

4- $y''=\frac{-2}{(x-1)^3}\cdot e^{-\frac{1}{x-1}}+\frac{1}{(x-2)^2}\cdot\frac{1}{(x-1)^2}\cdot e^{-1/x-1}$

$y''=\frac{-2(x-1)+1}{(x-1)^4}\cdot e^{-\frac{1}{x-1}}=\frac{-2x+3}{(x-1)^4}e^{-\frac{1}{x-1}}=0$ $y=1/e^{2}$

5- $x=0\Rightarrow y=e$

| $x$ | $-\infty$ | $-1$ | $0$ | $1$ | $3/2$ | $+\infty$ |
|---|---|---|---|---|---|---|
| $y'$ | | + | | ‖ | + | + |
| $y''$ | | + | | ‖ | + 0 | − |
| $y$ | 1 ↗ | $\sqrt{e}$ | $e$ | $e$ ‖ 0 ↗ | $1/e^{2}$ ↗ | 1 |




2) $y = x . e^{x}$ fonksiyonunun grafiğini çiziniz.

1) $(-\infty, +\infty)$

2) $\lim\limits_{x\to-\infty} y = -\infty . e^{-\infty} = -\infty . 0 \longrightarrow \frac{x}{e^{-x}} = \frac{1}{-e^{-x}} = -e^{x} = 0$

$\lim\limits_{x\to\infty} y = \infty . e^{\infty} = \infty . \infty = \infty$

$m = \lim\limits_{x\to\infty} \frac{y}{x} = \frac{x . e^{x}}{x} = \infty$

3) $y' = e^{x} + x . e^{x} = (1+x) . e^{x} = 0 \Rightarrow 1+x=0 \Rightarrow x=-1$

$x=-1$ için $y = \frac{1}{e}$

4) $y'' = e^{x} + (1+x) e^{x} = (2+x) e^{x} = 0 \Rightarrow 2+x=0 \Rightarrow x=-2$ Dönüm nok.

$x=-2 \Rightarrow y = \frac{-2}{e^{2}}$

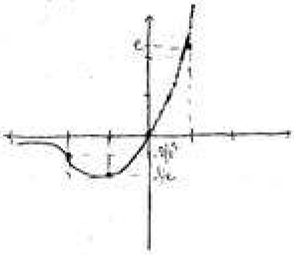
5) $x=0 \Rightarrow y=0$

$x=1 \Rightarrow y=e$

$x=-1 \Rightarrow y''>0$ minimum nokta

6)

| x | $-\infty$ | | $-2$ | | $-1$ | | $0$ | | $1$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y' | | − | | − | 0 | + | | | | + | |
| y'' | | − | 0 | + | | + | | | | + | |
| y | 0 | ↘ | $-2/e^{2}$ | ↘ | $-1/e$ min | ↗ | 0 | ↗ | e | ↗ | $+\infty$ |



Ö $y = x e^{-1/x}$

1) T.A $(-\infty, 0) \cup (0, +\infty)$

2) $\lim\limits_{x\to-\infty} y = -\infty . e^{-1/\infty} = -\infty$

$\lim\limits_{x\to+\infty} y = \infty . e^{-1/x} = +\infty$

$y = mx + n$ eğik asimptot

$m = \lim\limits_{x\to\mp\infty} \frac{y}{x} = \frac{\cancel{x} . e^{-1/x}}{\cancel{x}} = e^{-1/x} = e^{0} = 1$

$y = 1x + n \Rightarrow n = \lim\limits_{x\to\mp\infty} y - x = \lim\limits_{x\to\mp\infty} x . e^{-1/x} - x = \lim\limits_{x\to\mp\infty} x(e^{-1/x} - 1) = \infty.(1-1) = \infty.0$

$n = \lim\limits_{x\to\mp\infty} \frac{e^{-1/x} - 1}{\frac{1}{x}} = \frac{0}{0} \Rightarrow \lim\limits_{x\to\mp\infty} \frac{1/x^2 . e^{-1/x}}{-1/x^2} = -1 \quad n = -1$

$\boxed{y = x - 1}$

$\lim\limits_{x\to 0^+} (0+\varepsilon) . e^{-1/\varepsilon} = \lim\limits_{\varepsilon\to 0} \varepsilon e^{-1/\varepsilon} = 0 + 0 = 0$

$\lim\limits_{x\to 0^-} (0-\varepsilon) . e^{-1/x} = \lim\limits_{\varepsilon\to 0} -\varepsilon . e^{-1/x} = -\varepsilon . e^{\frac{1}{\varepsilon}} = 0 . \infty$

$\Downarrow$

$\lim\limits_{\varepsilon\to 0} \frac{-e^{1/\varepsilon}}{\frac{1}{\varepsilon}} = \frac{\infty}{\infty} \Rightarrow \lim\limits_{\varepsilon\to 0} \frac{1/\varepsilon^2 . e^{1/\varepsilon}}{-\frac{1}{\varepsilon^2}} = -\infty$

3) $y' = e^{-\frac{1}{x}} + x . \frac{1}{x^2} . e^{-1/x} = \underbrace{\frac{x^2 + x}{x^2} . e^{-1/x}} = 0$

$x.(x+1) = 0 \Rightarrow x = 0 \quad x = -1$'de Asimptotik durum $y = -e$

| $x$ | | $-1$ | | $0$ | |
|---|---|---|---|---|---|
| $x^2+x$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $x^2$ | $+$ | | $+$ | | $+$ |
| $y'$ | $+$ | | $-$ | | $+$ |

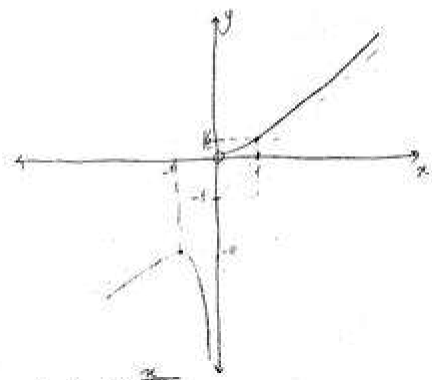
4. $y'' = \frac{2x+1 - 2x(x^2+x) . e^{-1/x}}{x^4} + \frac{x^2+x}{x^2} . \frac{1}{x^2} . e^{-\frac{1}{x}} = 0$

$y'' = \left[ \frac{2x^3 + x^2 - 2x^3 - 2x^2}{x^4} + \frac{x^2 + x}{x^4} \right] e^{-1/x} = 0 \qquad \frac{x}{x^4} e^{-1/x} = 0 \Rightarrow x = 0$

| $x$ | $-\infty$ | | $-1$ | | $0$ | | $1$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| $y'$ | | $+$ | $0$ | $-$ | ‖ | | $+$ | | $+$ |
| $y''$ | | $-$ | | $-$ | ‖ | $+$ | | | $+$ |
| $y$ | $-\infty$ $y=x-1$ | $\nearrow$ | $-e$ max | $\searrow$ | $-\infty$ ‖ $0$ | $\nearrow$ | $1/e$ | $\nearrow$ | $+\infty$ $y=x-1$ |

$x=1 \Rightarrow y=\frac{1}{e}$

$x=-1 \Rightarrow y=-e$




ii) $y=\frac{1}{2}(x-1)e^{\frac{x}{x-1}}$

1. T.A $(-\infty,1)\cup(1,+\infty)$

2. $\lim\limits_{x\to-\infty} y=-\infty$

$\lim\limits_{x\to+\infty} y=+\infty$

eğik asimptot

$\lim \frac{y}{x}=m=\lim\limits_{x\to\mp\infty}\frac{1}{2}\frac{x-1}{x}e^{\frac{x}{x-1}}=\frac{e}{2}$ $\quad y=mx+n \Rightarrow y=\frac{e}{2}x+n \to n=y-\frac{e}{2}x$

$n=\lim\limits_{x\to\mp\infty}\left[\frac{1}{2}(x-1)e^{\frac{x}{x-1}}-\frac{e}{2}x\right]=\lim\limits_{x\to\mp\infty}\frac{1}{2}(x-1)\left[e^{\frac{x}{x-1}}-e\frac{x}{x-1}\right]=\infty\cdot 0$

$\lim\limits_{x\to\mp\infty}\frac{1}{2}\frac{e^{\frac{x}{x-1}}-e\frac{x}{x-1}}{\frac{1}{x-1}}=\frac{e-e}{0}=\frac{0}{0}$

r.J

$$\lim_{x\to\pm\infty} \frac{\frac{1}{2}\left[\frac{1(x-1)-x}{(x-1)^2} e^{\frac{x}{x-1}} - e^{\frac{(x-1)-x}{(x-1)^2}}\right]}{-\frac{1}{(x-1)^2}} = -\frac{1}{2}\left[-e^{\frac{x}{x-1}} + e\right] = 0,$$

$$y = \frac{e}{2}x$$

$$\lim_{x\to 1^+} \frac{1}{2}(1+\varepsilon-1)e^{\frac{1+\varepsilon}{1+\varepsilon-1}} = \lim_{\varepsilon\to 0}\frac{1}{2}\varepsilon e^{\frac{1+\varepsilon}{\varepsilon}} = \frac{1}{2}0.e^{\infty} = 0\infty$$

$$\lim_{\varepsilon\to 0} \frac{\frac{1}{2}e^{\frac{1+\varepsilon}{\varepsilon}}}{\frac{1}{\varepsilon}} = \frac{\infty}{\infty} \Rightarrow$$

$$\lim_{\varepsilon\to 0}\frac{1}{2}\frac{\frac{\varepsilon-1(1+\varepsilon)}{\varepsilon^2}e^{\frac{1+\varepsilon}{\varepsilon}}}{-\frac{1}{\varepsilon^2}} = \lim_{\varepsilon\to 0}\frac{1}{2}\frac{\cancel{-1/\varepsilon^2}}{\cancel{-1/\varepsilon^2}}e^{\frac{1+\varepsilon}{\varepsilon}} = \frac{1}{2}e^{\infty} = \infty$$

$$\lim_{x\to 1^-} \frac{1}{2}.(1-\varepsilon-1).e^{\frac{1-\varepsilon}{1-\varepsilon-1}} = \lim_{\varepsilon\to 0} -\frac{\varepsilon}{2}e^{\frac{1-\varepsilon}{-\varepsilon}} = -0.e^{-\infty} = 0.0 = 0$$

3- $$y' = \frac{1}{2}\left[1.e^{\frac{x}{x-1}} + (x-1)\frac{-1}{(x-1)^2}e^{\frac{x}{x-1}}\right]$$

$$y' = \frac{1}{2}\left[\frac{(x-1)^2-(x-1)}{(x-1)^2}\right]e^{\frac{x}{x-1}} = \frac{1}{2}\left[\frac{x^2-2x+1-x+1}{(x-1)^2}\right].e^{\frac{x}{x-1}}$$

$$= \frac{1}{2}\left[x^2-3x+2\right] = \frac{1}{2}(x-1)(x-2) = 0 \Rightarrow$$

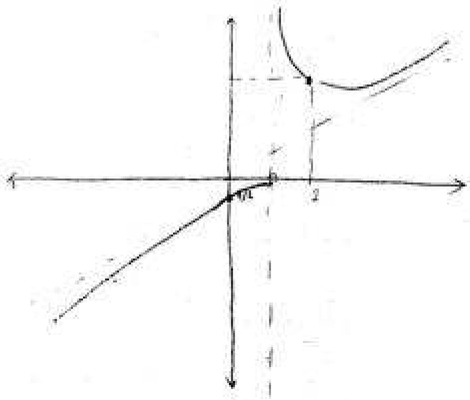
$x=2 \Rightarrow y = e^2/2$

$$y'' = \frac{1}{2}\left(\frac{(x-1)}{(x-1)^4}\right).e^{\frac{x}{x-1}} = 0 \quad \Rightarrow x=1 \quad \Rightarrow y=0$$

$x=0 \Rightarrow y = -\frac{1}{2}$

| $x$ | $-\infty$ | $0$ | $1$ | $2$ | $+\infty$ |
|---|---|---|---|---|---|
| $y'$ | $+$ | $+$ | $-$ | $0$ | $+$ |
| $y''$ | $-$ | $-$ | $0$ $+$ | | $+$ |
| $y$ | $-\infty \nearrow -1/2 \nearrow$ | $0,\ +\infty$ max | $\searrow$ | $e/2$ min | $\nearrow +\infty$ $y=e/2$ |

2 türev değer için pozitifse min



24.09.07/Perş.

## LOGARİTMİK FONKSİYONLAR

$y = \ln f(x) \qquad f(x) > 0 \qquad y \ln e = \ln f(x)$

$e^y = f(x) \rightarrow y' = e^{f(x)}$

Ö $y = \ln x \qquad x > 0 \qquad T.A\,(0, +\infty)$

$\lim\limits_{0^+} y = \lim\limits_{\varepsilon \to 0} \ln(0+\varepsilon) = \lim\limits_{\varepsilon \to 0} \ln \varepsilon = \ln 0 = -\infty$

$\lim\limits_{+\infty} y = \ln \infty = \infty$

$$m = \lim_{x \to +\infty} \frac{y}{x} = \lim_{x \to \infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{\frac{1}{x}}{1} = \frac{1}{\infty} = 0$$

$$y' = \frac{1}{x}$$

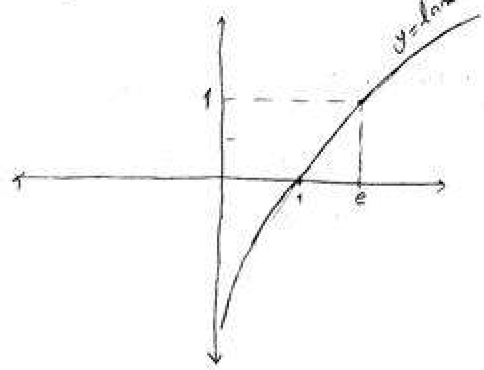
| | | 0 | |
|---|---|---|---|
| 1 | + | | + |
| x | − | | + |
| y' | − | | + |
| | | | T.A |

$$y'' = -\frac{1}{x^2} < 0 \quad \text{Konkav}$$

$x = 1 \Rightarrow y = 0$

$x = e \Rightarrow y = \ln e = 1$

| $x$ | 0 | | 1 | | $e$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $y'$ | | | | + | | + | |
| $y''$ | | − | | | | − | |
| $y$ | $-\infty$ | ↗ | 0 | ↗ | 1 | ↗ | $\infty$ |



6) $y = \ln(x+3)$ $\qquad x+3>0$

T.A $(-3, +\infty)$

$$\lim_{x\to -3^+} y = \lim_{\varepsilon\to 0} \ln(-3+\varepsilon+3) = \lim_{\varepsilon\to 0} \ln \varepsilon = \ln 0 = -\infty$$

$$\lim_{+\infty} y = \ln\infty = \infty$$

$$m = \lim_{x\to+\infty} \frac{y}{x} = \frac{\ln(x+3)}{x} \Rightarrow \frac{1}{\infty} = 0$$ eğik asimtot yok

$y' = \frac{1}{x+3}$ $\qquad x>-3 \Rightarrow y'>0$ artan

$y'' = -\frac{1}{(x+3)^2}$ $\qquad <0$ konkav

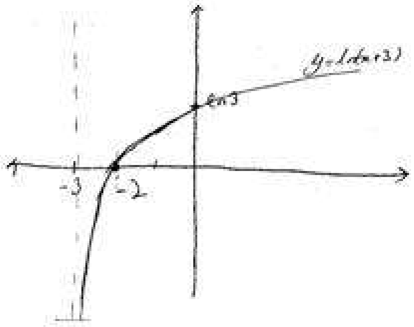
$y=0 \Rightarrow \ln 1 = \ln(x+3)$

$1 = x+3$

$x = -2$

$x=0 \Rightarrow y = \ln 3$

| $x$ | $-3$ | $-2$ | $0$ | $+\infty$ |
|---|---|---|---|---|
| $y'$ | $+$ | | $+$ | |
| $y''$ | $-$ | | $-$ | |
| $y$ | $-\infty \nearrow 0$ | $\nearrow \ln 3$ | $\nearrow +\infty$ | |

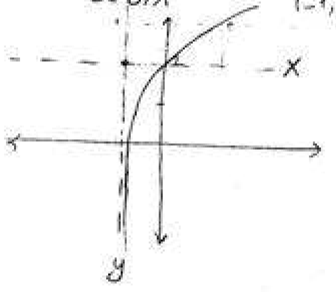


Ö. $y = 2 + \ln(x+1)$

$y - 2 = \ln(x+1)$

$y - 2 = \mathcal{Y} \rightarrow x + 1 = X$

$\mathcal{Y} = \ln X$




$(-1, +2) \rightarrow \mathcal{Y}$ ve $X$'in sıfır olduğu yer.

Ö. $y = \ln(x^2 - x - 2)$

$y = \ln[(x-2)(x+1)]$

| $x$ | $-\infty$ | $-1$ | | $2$ | $+\infty$ |
|---|---|---|---|---|---|
| $(x-2)$ | $-$ | | $-$ | $0$ | $+$ |
| $(x+1)$ | $-$ | $0$ | $+$ | | $+$ |
| $(x^2-x-2)$ | $+$ | | $-$ | | $+$ |
| | T.A | | | | T.A |

$T.A = (-\infty, -1) \cup (2, +\infty)$

$\lim\limits_{x \to -1^-} y = \lim\limits_{\varepsilon \to 0} \ln[(-1-\varepsilon)^2 - (-1-\varepsilon) - 2]$

$\lim\limits_{\varepsilon \to 0} \ln(1 + \varepsilon^2 + 2\varepsilon) + 1 + \varepsilon - 2 = \lim\limits_{\varepsilon \to 0} \ln(\varepsilon^2 + 3\varepsilon) = \ln 0 = -\infty$

$\lim\limits_{x \to 2^+} y = \lim\limits_{\varepsilon \to 0} \ln[(2+\varepsilon)^2 - (2+\varepsilon) - 2] = \lim\limits_{\varepsilon \to 0} \ln[4 + \varepsilon^2 + 4\varepsilon - 2 - \varepsilon - 2]$

$\lim\limits_{\varepsilon \to 0} \ln[\varepsilon^2 + 3\varepsilon] = \ln 0 = -\infty$

X

$$\lim_{x\to-\infty} y = \ln\infty = \infty$$

$$\lim_{x\to+\infty} y = \ln\infty = \infty$$

eğik asimtota bakılır.

$$m = \lim_{x\to\mp\infty} \frac{y}{x} = \lim_{x\to\mp\infty} \frac{\ln(x^2-x-2)}{x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\mp\infty} \frac{2x-1/x^2-x-2}{1} = \frac{1}{\infty} = 0 \rightarrow \text{eğik asimtot yok}$$

$$y' = \frac{2x-1}{x^2-x-2} \qquad 2x-1=0 \quad x=\frac{1}{2} \rightarrow \text{yerel extremum}$$

| $x$ | $-\infty$ | $-1$ | | $1/2$ | | $2$ | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $2x-1$ | $-$ | | $-$ | 0 | $+$ | | $+$ |
| $x^2-x-2$ | $+$ | 0 | $-$ | | $-$ | 0 | $+$ |
| $y'$ | $-$ | | $+$ | | $-$ | | $+$ |

$$y'' = \frac{2(x^2-x-2)-(2x-1)'(2x-1)}{(x^2-x-2)^2}$$

$$y'' = \frac{2x^2-2x-4-(4x^2-4x+1)}{(x^2-x-2)^2} = \frac{-2x^2+2x-5}{(x^2-x-2)^2} < 0$$

konkav

$\Delta = b^2-4ac = 4-4(-2)(-5) = -\dots$

$\Delta < 0$
kök yok

| $x$ | $-\infty$ | $-1{,}3$ | $-1$ | $1/2$ | $2$ | $2{,}3$ | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $y'$ | $-$ | | | $+$ 0 $-$ | | $+$ | |
| $y''$ | $-$ | | | $-$ $-$ | | $-$ | |
| $y$ | $+\infty$ ↘ | 0 | $-\infty$ | | $-\infty$ ↗ | 0 ↗ | $+\infty$ |

Tanımsız bölge

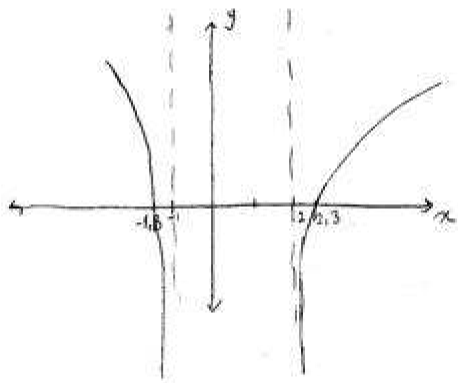
$y=0 \Rightarrow \ln 1 = 0$

$x^2-x-2=1$

$x^2-x-3=0$

$\Delta = 1+12 = 13$

$x_1 = \frac{1+\sqrt{13}}{2} = 2{,}3 \qquad \frac{1-\sqrt{13}}{2} = -1{,}3$




29.05.07
Cuma

Ö) $y = x - \ln(1+x)$ $\qquad 1+x > 0$

$\qquad x > -1$

T.A $(-1, +\infty)$

$\lim\limits_{x \to -1^+} y = \lim\limits_{x \to -1^+} (-1+\varepsilon) - \ln(1-1+\varepsilon) = \lim\limits_{\varepsilon \to 0} \left[(-1+\varepsilon) - \ln\varepsilon\right] = (-1 - \ln 0) = -1 - (-\infty) = +\infty$

$\lim\limits_{x \to +\infty} y = \infty - \ln\infty = \infty - \infty$

$= \lim\limits_{x \to +\infty} x \left[1 - \frac{\ln(1+x)}{x}\right] = \infty\left[1 - \frac{\infty}{\infty}\right] = \infty$

$\frac{\frac{1}{1+x}}{1} = \frac{1}{1+x} \Rightarrow \frac{1}{\infty} = 0$

$m = \lim\limits_{x \to \infty} \frac{y}{x} = \frac{x - \ln(1+x)}{x} = \lim\limits_{x \to \infty} \left(1 - \underbrace{\frac{\ln(1+x)}{x}}_{0}\right) \Rightarrow m = 1$

$y = mx + n \Rightarrow y = x + n \qquad n = \lim\limits_{x \to \infty} (y - x)$

$$n = \lim_{x \to \infty} \left[ x - \ln(1+x) - x \right] = -\infty$$

$y = x$

$\boxed{\text{m = 0 veya } \infty \text{ ise asimtot yoktur.}}$

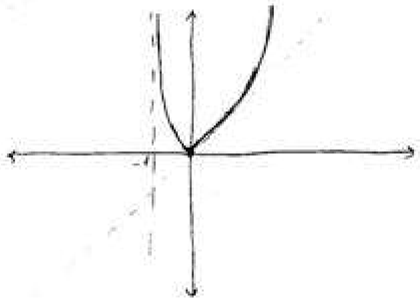
$$y' = 1 - \frac{1}{1+x} = \frac{1+x-1}{1+x} = \frac{x}{x+1}$$

| $x$ | $-\infty$ | $-1$ | | $0$ | | $+\infty$ |
|---|---|---|---|---|---|---|
| $x$ | $-$ | | $-$ | $0$ | $+$ | |
| $1+x$ | $-$ | $0$ | $+$ | | $+$ | |
| $\frac{x}{1+x}$ | $+$ | | $-$ | | $+$ | |
| | tanımsız | | $\searrow$ | | $\nearrow$ | |

$x = 0 \Rightarrow y = 0$

$$y'' = \frac{1.(1+x) - 1.x}{(1+x)^2} = \frac{1}{(1+x)^2} > 0 \quad \text{konveks}$$

| $x$ | $-1$ | | $0$ | | $+\infty$ |
|---|---|---|---|---|---|
| $y'$ | | $-$ | $0$ | $+$ | |
| $y''$ | | $+$ | | $+$ | |
| $y$ | $+\infty$ | $\searrow$ | $0$ min | $\nearrow$ | $+\infty$ |



Ö $y = x + \ln(x^2 + x - 2)$  $x^2 + x - 2 > 0$

$(x+2)(x-1) > 0$

| $-\infty$ | | $-2$ | | $1$ | | $+\infty$ |
|---|---|---|---|---|---|---|
| | $+$ | $0$ | $-$ | $0$ | $+$ | |
| | T.A | | | | T.A | |

T.A $(-\infty, -2) \cup (1, +\infty)$

$$\lim_{x \to \infty} y = +\infty$$

$$\lim_{x \to -\infty} = -\infty + \infty = \lim_{x \to -\infty} x\Big[1 + \underbrace{\frac{\ln(x^2+x-2)}{x}}_{\to 0}\Big] = -\infty[1] = -\infty$$

$$\lim_{x \to -2^-} y = -\infty$$

$$\lim_{x \to 1^+} y = -\infty$$

Sağdan soldan limitlere bakılır.

$$m = \lim_{x \to \infty} \frac{y}{x} = \lim_{x \to \infty} \frac{x + \ln(x^2+x-2)}{x} = \lim_{x \to \infty} 1 + \frac{\ln(x^2+x-2)}{x} = 1 + 0$$

$m = 1 \Rightarrow y = x + n \quad n = y - x$

$$n = \lim_{x \to \mp\infty} y - x = \lim_{x \to \mp\infty} \cancel{x} + \ln(x^2 + x - 2) - \cancel{x} = +\infty$$

$$y' = 1 + \frac{2x+1}{(x^2+x-2)} = \frac{x^2+x-2+2x+1}{x^2+x-2} = \frac{x^2+3x-1}{x^2+x-2}$$

$\Delta = b^2 - 4ac = 9 + 4 = 13$

$x_1 = \frac{-3+\sqrt{13}}{2} = 0{,}3 \quad x_2 = \frac{-3-\sqrt{13}}{2} = -3{,}3$

| $x$ | $-\infty$ | | $-3{,}3$ | | $-2$ | | $0{,}3$ | | $1$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x^2+3x-1$ | | $+$ | $0$ | $-$ | | $-$ | $0$ | $+$ | | $+$ | |
| $x^2+x-2$ | | $+$ | | $+$ | $0$ | $-$ | | $-$ | $0$ | $+$ | |
| $y'$ | | $+$ ↗ | | $-$ ↘ | | $+$ | | $+$ | | $+$ ↗ | |
| | | | max | | | | | | | | |

$$y'' = \frac{(2x+3)(x^2+x-2) - (2x+1)(x^2+3x-1)}{p^2}$$

$$y'' = \frac{\left[\cancel{2x^3}+2x^2-4x+3x^2+3x-6\right] - \left[\cancel{2x^3}+6x^2-2x+x^2+3x-1\right]}{p^2}$$

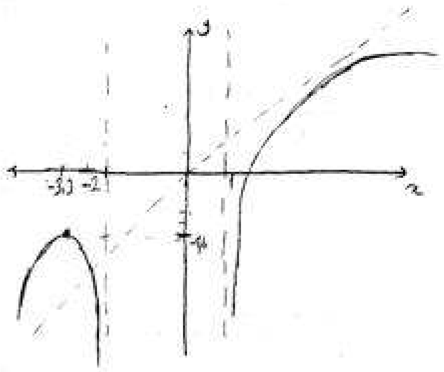
$$y'' = \frac{-2x^2-2x-5}{p^2} \longrightarrow \Delta = b^2-4ac$$

$$\Delta = 4-4(-2).(-5) = -36 < 0$$

dönüm nk. yok

kök yok
konkav

| $x$ | $-\infty$ | $-3{,}3$ | | $-2$ | | $0{,}3$ | | $1$ | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| $y'$ | $+$ | $0$ | $-$ | | | | | | $+$ |
| $y''$ | $-$ | | $-$ | | | | | | $-$ |
| $y$ | $-\infty$ ↗ | $-16$ | ↘ $-\infty$ | | | | | | ↗ $+\infty$ |



3/

$y = x + \frac{\ln x}{x}$ T.A $(0, +\infty)$

$\lim_{x \to 0^+} y = \lim_{\varepsilon \to 0} 0+\varepsilon \frac{\ln 0+\varepsilon}{0+\varepsilon} = \lim_{\varepsilon \to 0} \varepsilon + \frac{\ln \varepsilon}{\varepsilon} = 0 + \underbrace{\lim_{\varepsilon \to 0} \frac{\ln \varepsilon}{\varepsilon}}_{\lim \ln \varepsilon \cdot \frac{1}{\varepsilon}} = -\infty$

$\lim_{x \to +\infty} y = \infty + \frac{\infty}{\infty} = \infty + 0 = +\infty$

$m = \lim_{x \to \infty} \frac{y}{x} = \lim \frac{x + \frac{\ln x}{x}}{x} \Rightarrow 1 + \frac{\ln x}{x^2} = 1 \quad m = 1$

$m=1$ için
$y = x + n \Rightarrow n = y - x$

$\lim_{x \to \infty} y - x = \lim_{x \to \infty} \frac{\ln x}{x} = 0$ $\boxed{y = x}$ → eğik asimtot

$y = x + \frac{\ln x}{x}$

$y' = 1 + \frac{\frac{1}{x} \cdot x - 1 \cdot \ln x}{x^2} = \frac{x^2 + 1 - \ln x}{x^2}$

$x^2 + 1 - \ln x = 0$

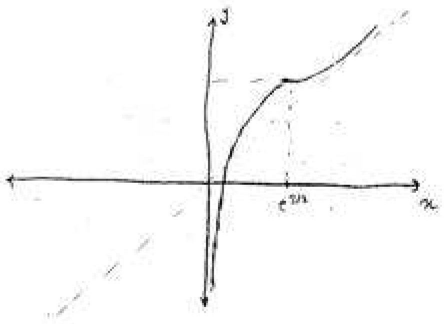
$x^2 + 1 = \ln x$

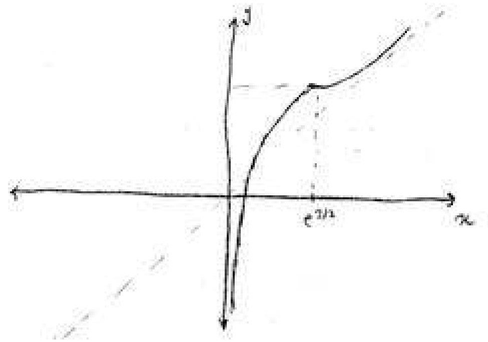
$\ln e^{x^2+1} = \ln x$

$e^{x^2+1} = x$

$y' = \ln e^{x^2+1} - \ln x = \ln \frac{e^{x^2+1}}{x} > 0$ konveks extremum nokta yok

$y'' = \frac{x(-3+2\ln x)}{x^4} =$ $x_1 = 0$
$x_2 = e^{3/2}$


y
x
$e^{3/2}$




31.05.07 / Perş

## —Belirli İntegral—

$$\int_a^b f(x)\,dx = g(x)\Big]_a^b = g(b) - g(a)$$

1- $[a,b]$ aralığında fonksiyon tanımlı olmalıdır.

Ö/ $\displaystyle\int_0^{\infty} \frac{dx}{x^2-x-6} = \int_0^{\infty} \frac{dx}{(x-3)(x+2)}$

fonksiyon $x=3$ ve $x=-2$'de tanımsızdır. Bu yüzden bu integralleri rezidü tekniğiyle çözeriz.

2- $\displaystyle\int_a^b f(x)\,dx$ → yaklaşıklık yöntemleri kullanılarak hesaplanabili

Ö/ $\displaystyle\int_0^{1/3} \frac{\sin x}{x}\,dx \Rightarrow \lim_{x\to 0} \frac{\sin x}{x} = 1$

$$= \int_0^{1/3} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)dx}{x} = \int_0^{1/3} \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!}\right)dx = x - \frac{x^3}{3\,3!} + \frac{x^5}{5\,5!}\Big|_0^{1/3} =$$

$$= \frac{1}{3} - \left(\frac{1}{3}\right)^3 / 3\cdot 3! + \left(\frac{1}{3}\right)^5 / 5\cdot 5! =$$

2. $\int_1^3 \ln x \, dx =$

$u = \ln x \qquad d\vartheta = dx$

$du = \frac{1}{x} dx \qquad \vartheta = x$

$$\int_1^3 \ln x \, dx = x \cdot \ln x \Big|_1^3 - \int_1^3 \frac{1}{x} x \, dx = x \cdot \ln x \Big|_1^3 - x \Big|_1^3$$

$$= 3\ln 3 - 1 \cdot \underbrace{\ln 1}_{0} - (3-1)$$

$$= 3\ln 3 - 2$$

Ö

$$\int_0^{\pi/2} \sin^2 x \, dx = \frac{1}{2} \int_0^{\pi/2} (1 - \cos 2x) \, dx = \frac{1}{2} \left[ x - \frac{1}{2} \sin 2x \right]_0^{\pi/2}$$

$$= \frac{1}{2} \left\{ \left( \frac{\pi}{2} - \frac{1}{2} \sin \pi \right) - \left( 0 - \frac{1}{2} \sin 0 \right) \right\} = \frac{\pi}{4}$$

Ö

$$\int_0^{\pi/2} \sin^8 x \, dx = ?$$

$$I_n = \int_0^{\pi/2} \sin^n x \, dx = \qquad (n > 0 \text{ ve } n \in Z^+)$$

$$= \int_0^{\pi/2} \sin^{n-1} x \cdot \sin x \, dx$$

$u = \sin^{n-1} x \qquad d\vartheta = \sin x \, dx \implies \vartheta = -\cos x$

$du = (n-1) \cos x \cdot \sin^{n-2} x \, dx$

$$= \int_0^{\pi/2} \sin^{n-1} x \sin x \, dx = -\cos x \cdot \sin^{n-1} x \Big|_0^{\pi/2} + (n-1) \int_0^{\pi/2} \cos x \cdot \cos x \cdot \sin^{n-2} x \, dx$$

$$0 + (n-1) \int_0^{\pi/2} \cos^2 x \sin^{n-2} x \, dx = (n-1) \int_0^{\pi/2} (1 - \sin^2 x) \sin^{n-2} x \, dx$$

$$= (n-1)\left[\int_0^{\pi/2} (\sin^{n-2} x\, dx) - \int_0^{\pi/2} \sin^{n} x\, dx\right]$$

$$\boxed{I_n = (n-1)\, I_{n-2} - (n-1)\, I_n}$$

$$\boxed{n I_n = (n-1) . I_{n-2}}$$

$$\boxed{I_n = \frac{n-1}{n} \cdot I_{n-2}}$$

$$I_8 = \int_0^{\pi/2} \sin^8 x\, dx = \frac{7}{8} I_6 = \frac{7}{8} \cdot \frac{5}{6} \cdot I_4 = \frac{7}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} I_2$$

$$= \frac{7}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} I_0 = \frac{7}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} =$$

$$I_0 = \int_0^{\pi/2} \sin^0 x\, dx = \int_0^{\pi/2} dx = x \Big|_0^{\pi/2} = \frac{\pi}{2}$$

$$I_8 = \int_0^{\pi/2} \sin^8 x\, dx = \frac{7.5.3.1}{8.6.4.2} \cdot \frac{\pi}{2}$$

$$\int_0^{\infty} x . e^{-x}\, dx =$$

$$\lim_{x\to 0} x . e^{-x} = 0$$

$$\lim_{x\to\infty} x . e^{-x} = \infty . 0 \longrightarrow \frac{x}{e^x} \Rightarrow \frac{\infty}{\infty} \overset{\text{L'hopital}}{\Rightarrow} \frac{1}{e^x} \Rightarrow \frac{1}{\infty} \Rightarrow 0$$

$x = u$ $\quad$ $dv = e^{-x} dx$

$dx = du$ $\quad$ $v = -e^{-x}$

$$\int_0^{\infty} x . e^{-x} dx = -x e^{-x}\Big|_0^{\infty} + \int_0^{\infty} e^{-x} dx = 0 - e^{-x}\Big|_0^{\infty} = -(e^{-\infty} - e^0) = 1$$

Ö $I_n=\int_0^{\infty} x^n \cdot e^{-x}dx \qquad (n\in Z^+)$

$u=x^n \Rightarrow du=nx^{n-1}dx$

$dv=e^{-x}dx \Rightarrow v=-e^{-x}$

$I_n=\int_0^{\infty} x^n e^{-x}dx=\cancel{-x^n e^{-x}}\Big|_0^{\infty}+n\int_0^{\infty} x^{n-1}e^{-x}dx$

$\boxed{I_n=n\,I_{n-1}}$

Ö $\int_0^{\infty} x^8 \cdot e^{-x}dx=8.I_7=8.7.I_6=8.7.6.I_5=8.7.6.5.4.3.2.1.I_0=$
$=8!.I_0$

$I_0=\int_0^{\infty} x^0 \cdot e^{-x}dx=\int_0^{\infty} e^{-x}dx=-e^{-x}\Big]_0^{\infty}=-(0-e^0)=1$

$I_8=8!$

$\boxed{I_n=\int_0^{\infty} x^n e^{-x}dx=n!} \qquad (n\in Z^+)$

Ö $I=\int_0^{\pi/2} e^{-x}\sin x dx=$

$u=\sin x \qquad dv=e^{-x}dx$

$du=\cos x\,dx \qquad v=-e^{-x}$

$I=\int_0^{\pi/2} e^{-x}\sin x dx=-e^{-x}\sin x\Big|_0^{\pi/2}+\int_0^{\pi/2} e^{-x}\cos x dx=$

$$= -\left(e^{-\pi/2}\sin\frac{\pi}{2} - e^{0}\sin 0\right) + \int_0^{\pi/2} e^{-x}\cos x\,dx$$ 38

$$= -e^{-\pi/2} + \int_0^{\pi/2} e^{-x}\cos x\,dx =$$

$u = \cos x$ $\qquad$ $e^{-x}dx = dv$

$du = -\sin x\,dx$ $\qquad$ $v = -e^{-x}$

$$= -e^{-\pi/2} - e^{-x}\cos x\Big|_0^{\pi/2} - \int_0^{\pi/2} e^{-x}\sin x\,dx = -e^{-\pi/2} + 1 - I = I$$

$$2I = 1 - e^{-\pi/2}$$

$$I = \frac{1 - e^{-\pi/2}}{2}$$

01.06.07
Cumo

Ö $\int_{-1}^{3} |x|\,dx =$

$x = 0$

| | $-\infty$ | 0 | $+\infty$ |
|---|---|---|---|
| $x$ | $-$ | 0 | $+$ |
| $\lvert x\rvert$ | $-x$ | | $+x$ |

$$\int_{-1}^{0} -x\,dx + \int_0^3 x\,dx = \frac{-x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_0^3 = -\left(0 - \frac{1}{2}\right) + \left(\frac{3^2}{2} - \frac{0^2}{2}\right)$$

$$= \frac{1}{2} + \frac{9}{2} = \frac{10}{2} = 5$$

Ö $\int_{-4}^{5} |x-1|\,dx =$ $\qquad$ $x - 1 = 0$, $x = 1$

| | $-\infty$ | 1 | $+\infty$ |
|---|---|---|---|
| $x-1$ | $-$ | 1 | $+$ |
| $\lvert x-1\rvert$ | $-(x-1)$ | 1 | $(x-1)$ |

$$\int_{-4}^{1} -(x-1)\,dx + \int_1^5 (x-1)\,dx = -\left(\frac{x^2}{2} - x\right)\Big|_{-4}^{1} + \frac{x^2}{2} - x\Big|_1^5$$

$$= -\left(\frac{1}{2} - 1\right) - \left(\frac{16}{2} + 4\right) + \left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right) = 20{,}5$$

$$\int_{-4}^{5} |(x^2 - x - 6)|\,dx \qquad \begin{matrix} x^2 - x - 6 = 0 \\ +2 \\ -3 \end{matrix} \rightarrow \begin{matrix} x = -2 \\ x = +3 \end{matrix}$$

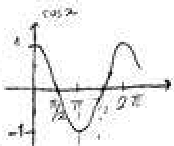
| | $-\infty$ | $-2$ | | $+3$ | | $+\infty$ |
|---|---|---|---|---|---|---|
| $x^2-x-6$ | $+$ | $0$ | $-$ | $0$ | $+$ | $+$ |
| $\lvert x^2-x-6\rvert$ | $x^2-x-6$ | | $-(x^2-x-6)$ | | $x^2-x-6$ | |

$$\int_{-4}^{-2}(x^2-x-6)\,dx - \int_{-2}^{3}(x^2-x-6)\,dx + \int_{3}^{5}(x^2-x-6)\,dx =$$

$$= \frac{x^3}{3} - \frac{x^2}{2} - 6x \Big|_{-4}^{-2} - \left(\frac{x^3}{3} - \frac{x^2}{2} - 6x\right)\Big|_{-2}^{3} + \frac{x^3}{3} - \frac{x^2}{2} - 6x \Big|_{3}^{5} =$$

$= \ldots\ldots$

$$\int_{0}^{2\pi} |\cos x|\,dx =$$




$$\int_{0}^{2\pi} |\cos x|\,dx = \int_{0}^{\pi/2} \cos x\,dx - \int_{\pi/2}^{3\pi/2} \cos x\,dx + \int_{3\pi/2}^{2\pi} \cos x\,dx =$$

$$= \sin x \Big|_{0}^{\pi/2} - \sin x \Big|_{\pi/2}^{3\pi/2} + \sin x \Big|_{3\pi/2}^{2\pi}$$

$$= \left(\sin\frac{\pi}{2} - \sin 0\right) - \left(\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right) + \left(\sin 2\pi - \sin\frac{3\pi}{2}\right) = 1 + 2 + 1 = 4$$

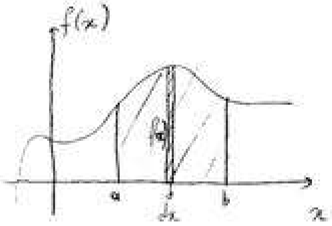
Ö: $\int_{-1}^{2} |x| . e^{-x} dx = \int_{-1}^{0} -x . e^{-x} dx + \int_{0}^{2} x e^{-x} dx =$

kısmi → $= -\left[ -x . e^{-x} \Big|_{-1}^{0} - e^{-x} \Big|_{-1}^{0} \right] + \left[ -x e^{-x} \Big|_{0}^{2} - e^{-x} \Big|_{0}^{2} \right] =$

$= -\left[ -(0+e) - (1-e) \right] + \left[ -(2 . e^{-2} - 0) - (e^{-2} - 1) \right] =$

$= \cancel{e} - 1 + \cancel{e} - 2e^{-2} - e^{-2} + 1 = 2 - 3e^{-2}$

ALAN




$ds = f(x) . dx$

$S = \int_{a}^{b} f(x) dx$

Ö: $f(x) = x . e^{-x}$ fonksiyonunun $x$ ekseni ile ve $x_1 = 1$, $x_2 = 2$ aralığındaki alanı bulunuz.

$[1, 2]$ aralığında $f(x)$ daima pozitiftir.

$S = \int_{1}^{2} x . e^{-x} dx =$

$S = -x . e^{-x} - e^{-x} \Big|_{1}^{2} = -2e^{-2} - e^{-2} - (-1e^{-1} - e^{-1}) = 2e^{-1} - 3e^{-2}$

Ö: $f(x) = x^2 - x - 2$ fonksiyonunun $(-4, +5)$ aralığında $x$ ekseni ile meydana getirdiği alanı hesaplayınız.

$(x-2)(x+1) \Rightarrow x = 2, x = -1$

$S = \int_{-4}^{-1} (x^2 - x - 2) dx + \int_{-1}^{2} (-x^2 - x - 2) dx + \int_{2}^{5} (x^2 - x - 2) dx = \ldots$

- $f(x)=x^2-x-2$ fonksiyonunun $x$ ekseni ile oluşturduğu alanı bulunuz

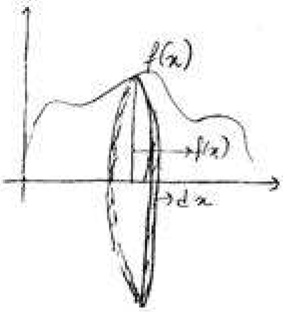
$$S=\left|\int_{-1}^{2}(x^2-x-2)\,dx\right| = \dots$$

$f(x)=x.e^{-x}$ [redacted] eğrisi ile $y=x$ doğrusu arasında kalan ($x=1$ & $x=2$ aralığında) alanı hesaplayınız

$$S'=\int_1^2 x\,dx=\left.\frac{x^2}{2}\right|_1^2=2-\frac{1}{2}=\frac{3}{2}$$

$$S=\left|\left(\frac{2}{e}-\frac{3}{e^2}\right)-\frac{3}{2}\right| \quad \text{—} \quad S.\ S'=S_1$$

HACİM




$$dV=\pi.f^2(x)\,dx$$

$$V=\pi\int_a^b f^2(x)\,dx$$

2. $f(x)=x.e^{-x}dx$ eğrisinin $[1,2]$ aralığındaki $x$ ekseni etrafında dönmesinden oluşan cismin hacmini bulunuz.

$$V=\pi\int_1^2 x^2.e^{-2x}dx=$$

$$\int x^2.e^{-2x}dx=$$

$x^2=u \qquad e^{-2x}dx=dV$

$2x\,dx=du \qquad -\frac{1}{2}e^{-2x}=V$

$$\int x^2e^{-2x}dx=-\frac{1}{2}x^2.e^{-2x}+\frac{1}{2}.2.\int \underline{x.e^{-2x}dx}$$

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$$\int x \cdot e^{-2x} dx = -\frac{1}{2} x \cdot e^{-2x} + \frac{1}{2} \int e^{-2x} dx = -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x}$$

$$\underbrace{\begin{matrix} u = x & e^{-2x} dx = d\vartheta \\ du = dx & -\frac{1}{2} e^{-2x} = \vartheta \end{matrix}}$$

$$\int x^2 e^{-2x} dx = -\frac{1}{2} x^2 \cdot e^{-2x} + \frac{1}{2} \cdot 2 \cdot \left( -\frac{1}{2} x \cdot e^{-2x} - \frac{1}{4} e^{-2x} \right)$$

$$= \left[ -\frac{1}{2} x^2 \cdot e^{-2x} - \left[ \frac{1}{2} x e^{-2x} + \frac{1}{4} e^{-2x} \right] \right]_1^2$$

Ö $f(x) = x^2 - x - 2$ $[-4, 3]$ aralığında hacmi hesaplayınız.

$$V = \pi \cdot \left[ \int_{-4}^{-1} (x^2 - x - 2)^2 dx + \int_{-1}^{2} (x^2 - x - 2)^2 dx + \int_{2}^{3} (x^2 - x - 2) dx \right] = \ldots$$