* ANALİZ II *
(Fizikçiler İçin)

Ders notlarını tutan Hanife BAŞ'a teşekkürler... (K.G.A)

- Periodik, çok yaklaşık periodik, kaotik
- Ani bir şekilde yok oluyor ve ortaya çıkıyor.

e EXPONANSİEL FONKSİYONLAR)

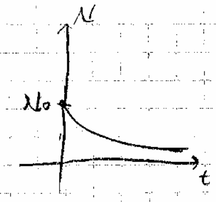
$\frac{3}{2} = 1,5$

$\frac{1}{9} = 0,\ldots\ldots \rightarrow$ RASYONEL SAYILAR

$\pi, \sqrt{2} \rightarrow$ İRRASYONEL SAYILAR

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e = 2.718282\ldots$$

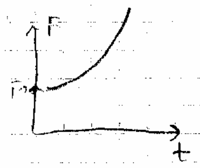
$N = N_0 . e^{-\lambda t} \rightarrow$



t	0	∞
N	N_0 $\rightarrow$	0

$F = F_0 . e^{+\alpha t}$

$\downarrow$



- TÜREVİ -

$y = f(x) = e^x$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x) = e^x \Rightarrow f(x+\Delta x) = e^{x+\Delta x} = e^x . e^{\Delta x}$$

$$e^{\Delta x} = \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{\Delta x} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n\Delta x} =$$

$$\lim_{n\to\infty}\Big[1 + \frac{1}{1!}\frac{1}{n}.n.\Delta x + 1.\frac{1}{2!}.\frac{1}{n^2}\underbrace{n\Delta x(n\Delta x - 1)}_{\Delta x^2 \cong 0} + \ldots\Big] =$$

$$\lim_{n\to\infty}\left[1 + \Delta x + 0 + 0 + 0\ldots\right] = e^{\Delta x} \cong 1 + \Delta x$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{e^x . e^{\Delta x} - e^x}{\Delta x} = \lim_{\Delta x\to 0}\frac{e^x(1+\Delta x) - e^x}{\Delta x} = \lim_{\Delta x\to 0}\frac{\cancel{e^x} + \Delta x e^x - \cancel{e^x}}{\cancel{\Delta x}}$$

$$= e^x //$$

* $y=f(x)=e^{x} \Rightarrow y'=\dfrac{dy}{dx}=f'(x)=e^{x}$

Ö $y=e^{x^4} \Rightarrow \dfrac{dy}{dx}=?$

$x^4=u \Rightarrow y=e^{u}$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=e^{u}\cdot 4x^3=4x^3\cdot e^{x^4}$$

$\dfrac{du}{dx}=4x^3$

$\dfrac{dy}{du}=e^{u}$

* Sonuç: $y=e^{u(x)} \Rightarrow y'=u'(x)\cdot e^{u(x)}$

Ö $y=e^{\cos 4x}$

$y'=-4\sin 4x\cdot e^{\cos 4x}$

Ö $y=e^{\operatorname{arctg} 3x^2}$

$y'=\dfrac{6x}{1+(3x^2)^2}\cdot e^{\operatorname{arctg} 3x^2}$

Ö $y=e^{\sin^3 2x}$

$y'=3\cdot 2\cdot\cos 2x\cdot\sin^2 2x\cdot e^{\sin^3 2x}=6\cdot\cos 2x\cdot\sin^2 2x\cdot e^{\sin^3 2x}$

! Ö $f=f_1\cdot f_2$

$\delta f=(f_1+\delta f_1)(f_2+\delta f_2)-f_1\cdot f_2$

$=\cancel{f_1\cdot f_2}+f_1\cdot\delta f_2+\delta f_1\cdot f_2+\underbrace{\cancel{\delta f_1\cdot\delta f_2}}_{0}-\cancel{f_1 f_2}$

$\dfrac{\delta f}{\delta x}=f_1\cdot\dfrac{\delta f_2}{\delta x}+\dfrac{\delta f_1}{\delta x}\cdot f_2 \qquad \left(\lim\limits_{\delta x\to 0}\dfrac{\delta f}{\delta x}=f'\right)$

$$\lim_{\delta x\to 0}\frac{\delta f}{\delta x}=\lim_{\delta x\to 0}f_1\cdot\frac{\delta f_2}{\delta x}+\lim_{\delta x\to 0}\frac{\delta f_1}{\delta x}\cdot f_2 \Rightarrow f'=f_1\cdot f_2'+f_1'\cdot f_2$$ *

Ö $y = x^3 \cdot e^{2x}$

$y' = 3x^2 \cdot e^{2x} + x^3 \cdot 2e^{2x} = (3x^2 + 2x^3)\, e^{2x}$

Ö $y = \operatorname{arctg} x^2 \cdot e^{\sin 2x}$

$y' = \frac{2x}{1+x^4} \cdot e^{\sin 2x} + \operatorname{arctg} x^2 \cdot 2 \cos 2x \cdot e^{\sin 2x}$

Ö $y = x \cdot \sin^2 x \cdot e^{x^3}$

$y' = (1 \cdot \sin^2 x + x\, 2 \sin x \cos x)\, e^{x^3} + x \sin^2 x \cdot 3x^2 \cdot e^{x^3}$

Ö $y = x \operatorname{arctg} x^3 e^{\sin^4 2x}$

$y' = \left(\operatorname{arctg} x^3 + \frac{3x^3}{1+x^6}\right) \cdot e^{\sin^4 2x} + e^{\sin^4 2x} \cdot 4 \cdot 2 \cos 2x \sin^3 2x \cdot x \operatorname{arctg} x^3$

Ö $y = e^{x^2}$ fonksiyonunun türev tanımına göre türevini bulunuz.

Çıkabilir !

$y + \delta y = e^{(x+\delta x)^2} = e^{x^2 + 2x\delta x + \delta x^2}$ $\qquad \delta x << 1 \Rightarrow \delta x^2 \cong 0$

$y + \delta y = e^{x^2 + 2x\delta x} = e^{x^2} \cdot e^{2x\delta x}$

$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$

$e^{2x\delta x} = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{2x\delta x n}$

$e^{2x\delta x} = \lim_{n\to\infty} \left[1 + \frac{2x\delta x n}{1!\, n} + \frac{2x\delta x n(2x\delta x n - 1)}{n^2 \cdot 2!} + \dots\right]$

$= \lim_{n\to\infty} \left[1 + 2x\delta x - \frac{2x\delta x n}{n^2} + \dots\right]$

$= 1 + 2x\delta x - 0, \dots$

$e^{2x\delta x} \cong 1 + 2x\delta x$

$\rightarrow y + \delta y = e^{x^2} \cdot 1 + 2x\delta x$

$\frac{\delta y}{\delta x} = \frac{(y+\delta y) - y}{\delta x} = \frac{e^{x^2}(1 + 2x\delta x) - e^{x^2}}{\delta x} = \frac{2x\delta x e^{x^2}}{\delta x} =$

$\frac{\delta y}{\delta x} = 2x e^{x^2} =$

$\lim \frac{\delta y}{\delta x} = \frac{dy}{dx} = 2x e^{x^2}$

$$y = \frac{u(x)}{v(x)}$$

$$y + \delta y = \frac{u+\delta u}{v+\delta v} = \frac{u+\delta u}{v+(1+\frac{\delta v}{v})} \longrightarrow \begin{matrix} \delta u << 1 & \delta v << 1 \\ \delta u^2 \cong 0 & \delta v^2 \cong 0 \end{matrix}$$

$$\frac{u+\delta u}{v} \cdot \left(1+\frac{\delta v}{v}\right)^{-1} \longrightarrow \varepsilon << 1 \text{ ve } \varepsilon^2 \cong 0 \text{ ise}$$

$$(1+\varepsilon)^n = 1 + \frac{n\varepsilon}{1!} + \frac{n(n-1)\varepsilon^2}{2!} + \frac{n(n-1)(n-2)\varepsilon^3}{3!} + \dots$$

$$(1+\varepsilon)^n \cong 1 + n.\varepsilon$$

$$y+\delta y \cong \frac{u+\delta u}{v} \cdot \left(1 - \frac{\delta v}{v}\right)$$

$$= \frac{(u+\delta u)(v-\delta v)}{v^2}$$

$$= \frac{uv + \delta u v - u\delta v - \cancel{\delta u \delta v}}{v^2} = \frac{uv + \delta u v - \delta v u}{v^2}$$

$$\frac{\delta y}{\delta x} = \frac{(y+\delta y)-y}{\delta x} = \frac{1}{\delta x} \cdot \left(\frac{uv+\delta u v - u\delta v}{v^2} - \frac{u}{v}\right)$$

$$= \frac{1}{\delta x}\left(\frac{\delta u v - u \delta v}{v^2}\right) = \frac{\frac{\delta u}{\delta x} v - u\frac{\delta v}{\delta x}}{v^2}$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\frac{\delta u}{\delta x} v - u \frac{\delta v}{\delta x}}{v^2}$$

$$\frac{dy}{dx} = \frac{\frac{du}{dx} v - u\frac{dv}{dx}}{v^2}$$

LOGARİTMA

* $\log_a a = 1$ * $\log_a a^x = x$

$$y = e^x \longrightarrow \log_e y = \ln y = \ln e^x = x \ln e = x \to x = \ln y$$

$$\to y = \ln x$$

* $\boxed{f(x) = e^x \Rightarrow f^{-1}(x) = \ln x}$

* $\ln f_1 . f_2 = \ln f_1 + \ln f_2$

* $\ln \frac{f_1}{f_2} = \ln f_1 - \ln f_2$

-TÜREVİ-

* $y = \ln x$

$x = e^{y}$

$\frac{dx}{dy} = e^{y} \Rightarrow \frac{dy}{dx} = e^{-y}$

* $\frac{dy}{dx} = \frac{1}{x}$

* $y = \ln x$ ve $x > 0 \Rightarrow y' = \frac{dy}{dx} = \frac{1}{x}$

Ö/ $y = \ln x$

	0	1	e	∞
$\ln x$	$-\infty \nearrow 0$	$\nearrow 1$	$\nearrow +\infty$	
y'		+		

① $y = \ln 0$

$y \ln e = \ln 0$

$\ln e^{y} = \ln 0$

$e^{y} = 0$

$x = 0 \Rightarrow y = -\infty$

② $y = \ln \infty$

$y \ln e = \ln \infty$

$\ln e^{y} = \ln \infty$

$e^{y} = \infty \Rightarrow y = \infty$

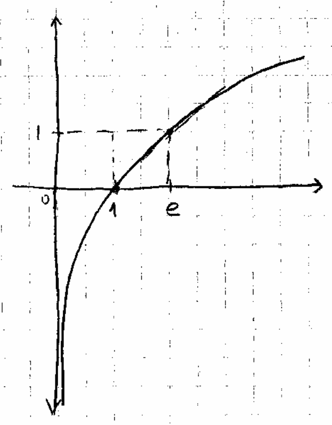
④ $x = e$ için

$y = \ln e$

$y = 1$

③ $x = 1$ için $y = \ln 1$

$e^{y} = 1 \Rightarrow y = 0$



Ö $y=\ln(x^2+1)$ $\quad x^2+1>0$

$y=\ln u \quad x^2+1=u \quad \Longrightarrow \quad \frac{dy}{dx}=2x \quad \frac{dy}{du}=\frac{1}{u}$

$$y'=\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{dy}{dx}=\frac{1}{u}\cdot 2x=\frac{2x}{x^2+1}$$

$*\ \boxed{y=\ln u \Rightarrow y'=\frac{u'}{u}}$

Ö $y=\ln(e^x+e^{-x})$

$$y'=\frac{e^x-e^{-x}}{e^x+e^x}$$

Ö $y=\ln(1+\mathrm{arctg}\,x^2)$

$$y'=\frac{\frac{2x}{1+x^4}}{1+\mathrm{arctg}\,x^2}=\frac{2x}{(1+x^4)\cdot(1+\mathrm{arctg}\,x^2)}$$

Ö $y=\ln(e^{\sin 2x}+x)$

$$\frac{2\cos 2x\cdot e^{\sin 2x}+1}{e^{\sin 2x}+x}$$

Ö $y=e^{x^2}\cdot\ln(x+1)$

$$y'=2x\cdot e^{x^2}\cdot\ln(x+1)+e^{x^2}\cdot\frac{1}{x+1}$$

Ö $y=\frac{1+\ln(e^x+1)}{e^{x^2}}$

$$y'=\frac{\frac{e^x}{e^x+1}\cdot e^{x^2}-2xe^{x^2}[1+\ln(e^x+1)]}{e^{2x^2}}$$

ÜSTEL FONKSİYONLAR

$y = f(x)^{g(x)}$

— TÜREVİ —

$\ln y = \ln f(x)^{g(x)} = g(x) \cdot \ln f(x)$

$\frac{y'}{y} = g'(x) \cdot \ln f(x) + g(x) \cdot \frac{f'(x)}{f(x)}$

$y' = f^{g}\left[g' \ln f + g \cdot \frac{f'}{f}\right]$

Ö $y = (1+x^3)^{\sin 2x}$

$\ln y = \ln (1+x^3)^{\sin 2x} = \sin 2x \ln (1+x^3)$

$\frac{y'}{y} = 2\cos 2x \cdot \ln (1+x^3) + \sin 2x \cdot \frac{3x^2}{1+x^3}$

$y' = (1+x^3)^{\sin 2x}\left[2\cos 2x \ln (1+x^3) + \frac{3x^2 \sin 2x}{1+x^3}\right]$

Ö $y = \left(1+\operatorname{arctg}^3(x^2+1)\right)^{\sin^2 x}$

$\ln y = \sin^2 x \ln\left(1+\operatorname{arctg}^3(x^2+1)\right)$

$\frac{y'}{y} = 2\sin x \cos x \cdot \ln (1+\operatorname{arctg}^3(x^2+1)) + \sin^2 x \frac{3\operatorname{arctg}^2(x^2+1) \cdot \frac{2x}{1+(x^2+1)^2}}{1+\operatorname{arctg}^3(x^2+1)}$

$y' = \left(1+\operatorname{arctg}^3(x^2+1)\right)^{\sin^2 x} \cdot 2\sin x \cdot \cos x \ln (1+\operatorname{arctg}^3(x^2+1)) + \sin^2 x \frac{3\operatorname{arctg}(x^2+1) \cdot \frac{2x}{1+(x^2+1)^2}}{1+\operatorname{arctg}^3(x^2+1)}$

Ö $y = (1+\operatorname{arctg} x)^{x^3+1}$ eğrisinin $x=1$ noktasındaki teğet denklemini bulunuz.

$x_0 = 1 \quad y_0 = (1+\operatorname{arctg} 1)^2 = \left(1+\frac{\pi}{4}\right)^2$

$\operatorname{arctg} 1 = u$

$\operatorname{tg} u = 1$

$u = \frac{\pi}{4}$

DEVAMI ARKADA!

$\ln y = (x^3+1) \cdot \ln(1+\operatorname{arctg} x)$

$\frac{y'}{y} = 3x^2 \cdot \ln(1+\operatorname{arctg} x) + (x^3+1) \frac{1/1+x^2}{1+\operatorname{arctg} x}$

$y' = y_0 \cdot \left[3\ln\left(1+\frac{\pi}{4}\right) + \frac{1}{\left(1+\frac{\pi}{4}\right)} \right]$

$T.D = y - y_0 = m(x - x_0)$

Ö $y = (1+x)^{x^x}$

$\ln y = x^x \cdot \ln(1+x)$

$\frac{y'}{y} = (x^x)' \cdot \ln(1+x) + x^x \cdot \frac{1}{1+x}$

$y' = (1+x)^{x^x} \left[x^x \cdot (1+\ln x) \cdot \ln(1+x) + \frac{x^x}{1+x} \right]$

$x^x = u$

$\ln u = x \cdot \ln x$

$\frac{u'}{u} = \ln x + x \cdot \frac{1}{x}$

$u' = x^x(1+\ln x)$

Ö $y = (1+e^{x^2})^{(x+1)}$ eğrisinin $x=0$ noktasındaki $T.D = ?$

$x = 0 \Rightarrow y = (1+e^0)^1 = 2$

$\ln y = (x+1) \cdot \ln(1+e^{x^2})$

$\frac{y'}{y} = 1 \cdot \ln(1+e^{x^2}) + (x+1) \cdot \frac{2x \cdot e^{x^2}}{1+e^{x^2}}$

$y_0' = m = y_0 (\ln 2 + 0) = 2 \cdot \ln 2 = \ln 4$

$y - y_0 = m \cdot (x - x_0)$

$y - 2 = \ln 4 \cdot (x - 0)$

$y = x \cdot \ln 4 + 2$

Ö $y=(1+\sin^2 x)^{x+1}$ $x=0$ nok. T.D=?

$x=0 \Rightarrow y=1$

$\ln y = (x+1) . \ln(1+\sin^2 x)$

$\frac{y'}{y} = \ln(1+\sin^2 x) + (x+1) . \frac{2\cos x . \sin x}{1+\sin^2 x}$

$y' = 1 . \left(\ln(1+\sin^2 0) + (0+1) . \frac{2 . \cos 0 . \sin 0}{1+\sin^2 0}\right)$

$y'=0$

$y-y_0=0.$

$y-1=0$

$\boxed{y=1}$

Ö $y=\left(e^{2x}+e^{-2x}\right)^{(x^2+1)}$ $x=0$ nok. T.D

$x=0 \Rightarrow y=(1+1)^1=2$ $(0,2)$

$\ln y = (x^2+1) . \ln(e^{2x}+e^{-2x})$

$\frac{y'}{y} = 2x . \ln(e^{2x}+e^{-2x}) + (x^2+1) . \frac{2 . e^{2x} - 2 . e^{-2x}}{e^{2x}+e^{-2x}} =$

$y' = \left(e^{2x}+e^{-2x}\right)^{(x^2+1)} . 2x . \ln(e^{2x}+e^{-2x}) + (x^2+1) . \frac{2e^{2x}-2 . e^{-2x}}{e^{2x}+e^{-2x}}$

$m=y'=2 . 0 = 0$

$y-y_0 = \underset{0}{m} . (x-x_0)$

$y-2=0$

$\boxed{y=2}$

$* \; y=e^{f(x)} \Rightarrow y' = f'(x) . e^{f(x)}$ $* \; y=\ln f(x) \Rightarrow y' = \frac{f'(x)}{f(x)}$

$* \; y=f(x)^{g(x)} \Rightarrow \ln y = g(x) \ln f(x)$

$\Rightarrow y' = y . \left[g' \ln f + g \frac{f'}{f}\right] = f^{g}\left[g' \ln f + g \frac{f'}{f}\right]$

KUVVET SERİLERİ

$$f(x)=\sum_{n=0}^{\infty} a_n x^n$$

$$f(x)=a_0+a_1x+a_2x^2+a_3x^3+\dots a_nx^n+\dots$$

$x=0 \Rightarrow f(0)=a_0+0$

$* \; a_0=f(0)$

$$f'(x)=a_1+2a_2x+3a_3x^2+4a_4x^3+\dots n.a_n.x^{n-1}+\dots$$

$x=0 \Rightarrow f'(0)=a_1+0$

$f'(0)=a_1$

$$f''(x)=2.1a_2+3.2.1.a_3x+4.3.a_4x^2+\dots+n.(n-1)a_nx^{n-2}$$

$f''(0)=2!a_2+0$

$$a_2=\frac{f''(0)}{2!}$$

$$f'''(x)=3!a_3+4.3.2.1a_4x+\dots+n(n-1)(n-2).x^{n-3}+\dots$$

$f'''(0)=3!a_3+0$

$$a_3=\frac{f'''(0)}{3!} \qquad a_4=\frac{f^{(IV)}(0)}{4!} \qquad a_5=\frac{f^{(V)}(0)}{5!} \quad \dots$$

$$* \; a_n=\frac{f^{(n)}(0)}{n!}$$

$$* \; f(x)=f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+\dots+\frac{f^{(n)}(0)}{n!}x^n+\dots$$

NOT = Fonksiyonun türevleri $x=0$'da tanımlı olmalıdır.

Ö. $f(x)=e^x$

$f(0)=e^0=1$

$f'(x)=e^x$

$f'(0)=1$

$f''(x)=e^x \Rightarrow f''(x)=1 \quad \Rightarrow f''(0)=1$

$$e^x=1+x+\frac{1}{2!}x^2+\frac{1}{3!}.x^3+\dots\frac{1}{n!}x^n+\dots$$

Ö $f(x)=\sin x$

$f(0)=0$

$f'(x)=\cos x \quad f'(0)=1$

$f''(x)=-\sin x \quad f''(0)=0$

$f'''(x)=-\cos x \quad f'''(0)=-1$

$f^{IV}(x)=\sin x \quad f^{IV}(0)=0$

$f^{V}(x)=\cos x \quad f^{V}(0)=1$

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}\dots$$

Ö $f(x)=\cos x$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots$$

$$\frac{d^2\theta}{dt^2}=-\frac{g}{\ell}\sin\theta$$

$$\sin\theta=\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}+\dots \qquad \theta \ll 1$$

$\sin\theta \simeq \theta$

$$\frac{d^2\theta}{dt^2}\simeq-\frac{g}{\ell}\theta \Rightarrow \omega^2=\frac{g}{\ell} = \omega=\sqrt{\frac{g}{\ell}}$$

$$\frac{2\pi}{T}=\sqrt{\frac{g}{\ell}} \Rightarrow T=2\pi\sqrt{\frac{\ell}{g}}$$

* $f(x)=f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+\dots+\frac{f^{n}(0)x^n}{n!}$ serisine Maclaurin serisi denir.

$x=a \rightarrow f(x)=a_0+a_1(x-a)+a_2(x-a)^2+a_3(x-a)^3+\dots a_n(x-a)^n+\dots$

$$f(x)=\sum_{n=0}^{\infty} a_n(x-a)^n$$

$f(a)=a_0$

$f'(a)=1+2a_2(x-a)+3a_3(x-a)^2+\dots na_n(x-a)^{n-1}+\dots$

$f(a)=a_1+0 \Rightarrow f'(a)=a_1$

$$f''(x) = 1\cdot 2a_2 + 3\cdot 2\cdot 1 a_3 (x-a) + \ldots + n(n-1)(x-a)^{n-2} + \ldots$$

$$f''(a) = 2!\, a_2 + 0 \qquad a_2 = \frac{f''(a)}{2!}$$

$$f'''(x) = 3!\, a_3 + 4\cdot 3\cdot 2(x-a) + 5\cdot 4\cdot 3(x-a)^2 + \ldots n\cdot(n-1)\cdot(n-2)(x-a)^{n-3}$$

$$f'''(a) = 3!\, a_3 \quad \Rightarrow a_3 = \frac{f'''(a)}{3!}$$

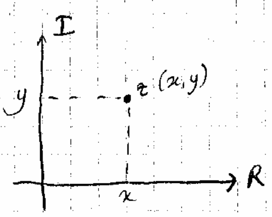
$$* \; f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \ldots \frac{f^n(a)(x-a)^n}{n!}$$

serisine Taylor serisi denir.

Ö: $f(x) = e^x \to x = 0,1$ taylor

$$e^x = e^{0,1} + \frac{e^{0,1}(x-0,1)}{1!} + \frac{e^{0,1}(x-0,1)^2}{2!} + \frac{e^{0,1}(x-0,1)^3}{3!} + \ldots$$

EK BİLGİ: KARMAŞIK SAYILAR



$z = x + iy \qquad i = \sqrt{-1}$

Toplam:

$z_1 = 2-i$
$z_2 = 1+3i$

$$2z_1 - 3z_2 = 2(2-i) - 3(1+3i)$$
$$= 4 - 2i - 3 - 9i$$
$$= 1 - 11i$$

Çarpım:

$z_1 = 2-i$
$z_2 = 1+3i$

$$(3z_1)(2z_2) = 6(z_1 . z_2)$$
$$z_1 . z_2 = (2-i).(1+3i) = 2 + 6i - i + 3 = 5 + 5i$$
$$6(z_1 . z_2) = 30 + 3i$$

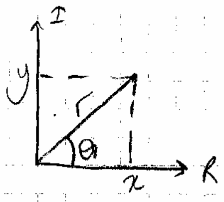
Bölüm:

$$\frac{z_1}{z_2} = \frac{2-i}{1+3i} \Longrightarrow \frac{z_1}{z_2} = \frac{z_1 . z_2^*}{z_2 . z_2^*} = \frac{-1-7i}{10} = -\frac{1}{10} - \frac{7}{10} i$$

$$z_2 = 1+3i \Rightarrow \underset{\substack{\downarrow \\ \text{eşleniği}}}{z_2^*} = 1-3i$$

$$z_2 . z_2^* = (1+3i)(1-3i)$$
$$= 1+9 = 10$$

$$z_1 . z_2^* = (2-i)(1-3i)$$
$$= -1-7i$$



$$z = x+iy$$
$$\left.\begin{matrix} x = r.\cos\theta \\ y = r.\sin\theta \end{matrix}\right\} z = r(\cos\theta + i\sin\theta)$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \frac{\theta^8}{8!} \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$\cos\theta + i\sin\theta = 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} - \frac{\theta^6}{6!} - i\frac{\theta_7}{7!} + \frac{\theta^8}{8!} + \dots$$

* $i^2 = -1, \quad i^3 = -i, \quad i^4 = 1, \quad i^5 = i \quad i^6 = -1$

$$\cos\theta + i\sin\theta = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \frac{(i\theta)^6}{6!} + \frac{(i\theta)^7}{7!} + \frac{(i\theta)^8}{8!} \dots$$

NOT: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \dots$

$$\cos\theta + i\sin\theta = e^{i\theta}$$

$$\boxed{\begin{matrix} e^{i\theta} = \cos\theta + i\sin\theta \\ e^{-i\theta} = \cos\theta - i\sin\theta \end{matrix}}$$

Ö. $(\cos\theta + i\sin\theta)^2 = (e^{i\theta})^2 = e^{i\theta} . e^{i\theta} = e^{2i\theta} = \cos 2\theta + i\sin 2\theta$

$$\cos^2\theta - \sin^2\theta + i\, 2\cos\theta\sin\theta = \cos 2\theta + i\sin 2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin 2\theta = 2\cos\theta . \sin\theta$$

$(\cos\theta + i\sin\theta)^3 = e^{3i\theta} = \cos 3\theta + i\sin 3\theta$

$\cos^3\theta + 3\cos^2\theta(i\sin\theta) + 3\cos\theta(i\sin\theta)^2 + (i\sin\theta)^3 =$

$= \cos^3\theta - 3\cos\theta\sin^2\theta + i(3\cos^2\theta\sin\theta - \sin^3\theta) = \cos 3\theta + i\sin 3\theta$

$\cos 3\theta = \cos^3\theta - 3\cos\theta . \sin^2\theta = \cos^3\theta - 3\cos\theta(1-\cos^2\theta)$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ //

$\sin 3\theta = 3\cos^2\theta . \sin\theta - \sin^3\theta$

$\sin 3\theta = 3.(1-\sin^2\theta) . \sin\theta - \sin^3\theta$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ //

$e^{i(\theta_1+\theta_2)} = \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$

$e^{i\theta_1} . e^{i\theta_2} = (\cos\theta_1 + i\sin\theta_1).(\cos\theta_2 + i\sin\theta_2)$

$= (\cos\theta_1 . \cos\theta_2 - \sin\theta_1 . \sin\theta_2) + i(\sin\theta_1 . \cos\theta_2 + \sin\theta_2 . \cos\theta_1)$

$\cos(\theta_1+\theta_2) = \cos\theta_1 . \cos\theta_2 + \sin\theta_1 . \sin\theta_2$ //

$\sin(\theta_1+\theta_2) = \sin\theta_1 . \cos\theta_2 + \sin\theta_2 . \cos\theta_1$ //

$e^{i\theta} + e^{-i\theta} = 2\cos\theta \implies \boxed{\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}}$

$e^{i\theta} - e^{i\theta} = i2\sin\theta \implies \boxed{\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}}$

$\cos^2\theta = \dfrac{e^{2i\theta}+2+e^{-2i\theta}}{4}$

$\sin^2\theta = \dfrac{e^{2i\theta}-2+e^{-2i\theta}}{-4} = \dfrac{-e^{2i\theta}+2-e^{-2i\theta}}{4}$

$\cos^2\theta + \sin^2\theta = \frac{1}{4}(4) = 1$ //

$z^2 = 1-i \quad ; \quad \sqrt{1-i} = ?$

$x = 1 \rightarrow y = -1 \qquad r = \sqrt{x^2+y^2} = \sqrt{2}$

$\tan\theta = \frac{y}{x} \qquad \theta = \operatorname{arctg}\left(\frac{y}{x} + \delta\right)$

$tg\theta = -1$

$tg\theta = -1 \Rightarrow \quad \theta_1 = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$\theta_2 = \frac{3\pi}{2} + \frac{\pi}{4} = \frac{7\pi}{4}$

$x = \sqrt{2}.\cos\theta \rightarrow \cos\ (+)$ olmalı $(x=1)$

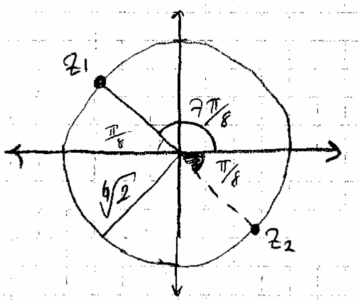
$y = \sqrt{2}.\sin\theta \rightarrow \sin\ (-)$ olmalı $(y=-1)$

$\boxed{\theta_1 = \frac{7\pi}{4}} \qquad \theta_2 = \frac{7\pi}{4} + 2\pi = \boxed{\frac{15\pi}{4}}$

$$z^2 = 1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = \sqrt{2}\, e^{i\frac{7\pi}{4}}$$

$$z_1 = \sqrt{1-i} = \sqrt[4]{2}\cdot e^{i\frac{7\pi}{8}} = \sqrt[4]{2}\left(\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}\right)$$

$$z_2 = \sqrt{1-i} = \sqrt[4]{2}\, e^{i\frac{15\pi}{8}} = \sqrt[4]{2}\left(\cos\frac{15\pi}{8} + i\sin\frac{15\pi}{8}\right)$$



Ödev $z^3 = 1+2i$ (1) $\quad z^6 = -1$ (2) $\quad z^4 = i$ (3)

$$\boxed{(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta = e^{in\theta}}$$

$\Downarrow$

MOIVRE FORMÜLÜ

HYPERBOLİK FONKSİYONLAR

$$\sinh x = \frac{e^{x}-e^{-x}}{2}$$

$$\cosh x = \frac{e^{x}+e^{-x}}{2}$$

$$\operatorname{tgh} x = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

$$\operatorname{cotgh} x = \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

$$\operatorname{sech} x = (\cosh x)^{-1} = \frac{2}{e^{x}+e^{-x}}$$

$$\operatorname{cosech} x = (\sinh x)^{-1} = \frac{2}{e^{x}-e^{-x}}$$

Ö. $\cosh^2 x = \dfrac{e^{2x}+2+e^{-2x}}{4}$

$$\sinh^2 x = \frac{e^{2x}-2+e^{-2x}}{4}$$

$$\cosh^2 x - \sinh^2 x = \frac{4}{4} = 1$$

Ö. $\cosh x + \sinh x = e^{x}$

$$\cosh x - \sinh x = e^{-x}$$

– TÜREVİ –

$$(\sinh x)' = \frac{e^{x}+e^{-x}}{2} = \cosh x$$

$$(\cosh x)' = \frac{e^{x}-e^{-x}}{2} = \sinh x$$

Ö. $\sinh 0 = 0$

Ö. $\cosh 0 = 1$

$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!}$

$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} \ldots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots$

$\sin(ix) = ix - \frac{(ix)^3}{3!} + \frac{(ix)^5}{5!} - \frac{(ix)^7}{7!} = ix + \frac{ix^3}{3!} + \frac{ix^5}{5!} + \frac{ix^7}{7!}$

$= i\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \ldots\right) = i\sinh x$

$\boxed{\sin(ix) = i\sinh x}$

$\cos x = \frac{e^{ix} + e^{-ix}}{2}$

$-\cos ix = \frac{e^{i(ix)} + e^{-i(ix)}}{2} = \frac{e^{-x} + e^{x}}{2} = \frac{e^{x} + e^{-x}}{2} = \cosh x$

$\boxed{\cos ix = \cosh x}$

L'HOSPITAL TEOREMI

limit işlemlerinde faydalı olan ve $\frac{0}{0}$ belirsizliklerinde kullanılan bir tekniktir. Tabii ki $\frac{0}{0}$ belirsizliğine düşürülebilen durumlarda da kullanılan bir tekniktir.

$\lim_{x \to 0} \frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0}$

$\lim_{x \to 0} \frac{x - x^3/3! + \frac{x^5}{5!} - x^7/7! + \ldots}{x} = \lim_{x \to 0} \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} \ldots\right)$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)} = \frac{0}{0}$ Belirsiz

$$\frac{f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots}{g(a) + \frac{g'(a)}{1!}(x-a) + \frac{g''(a)}{2!}(x-a)^2 + \frac{g'''(a)}{3!}(x-a)^3 + \ldots} = \frac{0+0+0+}{0+0+0} = \frac{0}{0}$$

DEVAMI ARKADA!

$$\frac{f(a)+(x-a)\left[\frac{f'(a)}{1!}+\frac{f''(a)}{2!}(x-a)+\frac{f'''(a)}{3!}(x-a)^2\cdots\right]}{g(a)+(x-a)\left[\frac{g'(a)}{1!}+\frac{g''(a)}{2!}(x-a)+\frac{g'''(a)}{3!}(x-a)^2\cdots\right]}=$$

$$=\frac{0+f'(a)+0+0}{g'(a)+0+0}=$$

$$\boxed{\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{f(a)}{g(a)}=\frac{f'(a)}{g'(a)}=\frac{f''(a)}{g''(a)}}$$

$$\lim_{x\to 0}\frac{\sin x}{x}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\cos x}{1}=\frac{\cos 0}{1}=1$$

$$\lim_{x\to 0}\frac{x^2}{1-\cos x}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{2x}{\sin x}=\frac{0}{\sin 0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{2}{\cos x}=\frac{2}{\cos 0}=\frac{2}{1}=2$$

$$\lim_{x\to 0}\frac{e^x-e^{-x}}{x^2}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{e^x+e^{-x}}{2x}=\frac{\cancel{2}}{\cancel{2}x}=\frac{1}{x}=\frac{1}{0}=\infty$$

1) $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\infty}{\infty}\Rightarrow\lim_{x\to a}\frac{f'(x)}{g'(x)}$

2) $\lim_{x\to a}\left[f'(x)\mp g'(x)\right]=\infty-\infty$

$$\lim_{x\to a}\left(\frac{1}{\frac{1}{f}}-\frac{1}{\frac{1}{g}}\right)=\lim_{x\to a}\left(\frac{\frac{1}{g}-\frac{1}{f}}{\frac{1}{fg}}\right)=\frac{0}{0}\Rightarrow\lim_{x\to a}\frac{\left(\frac{1}{g}-\frac{1}{f}\right)'}{\left(\frac{1}{fg}\right)'}$$

3) $\lim\limits_{x\to a} f(x)g(x) = 0.\infty$

$$\lim_{x\to a} \frac{f}{1/g} = \frac{0}{\frac{1}{\infty}} = \frac{0}{0} \Rightarrow \lim_{x\to a} \frac{f'}{\left(\frac{1}{g}\right)'}$$

4) 0^0, ∞^0, 1^∞

$\lim\limits_{x\to a} f(x)^{g(x)}$, 0^0 veya ∞^0 veya 1^∞

$$\lim_{x\to a} g(x)\ln f(x) \begin{cases} \to 0\ln 0 = 0.\infty \\ \to 0.\ln\infty = 0.\infty \\ \to \infty.\ln 1 = \infty.0 \end{cases}$$

$$\lim_{x\to a} \frac{\ln f(x)}{\left(\frac{1}{g}\right)} \Rightarrow \lim_{x\to a} \frac{(\ln f)'}{\left(\frac{1}{g}\right)'}$$

Ö $\lim\limits_{x\to 0} \frac{\ln(1+x^2)}{x} = \frac{\ln 1}{0} = \frac{0}{0}$

$$\lim_{x\to 0} \frac{\frac{2x}{1+x^2}}{1} = 0$$

Ö $\lim\limits_{x\to\infty} \frac{\ln(1+x^2)}{x^2} = \frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{\frac{2x}{1+x^2}}{2x} = \frac{1}{1+x^2} = \frac{1}{\infty} = 0$$

$$\boxed{\lim_{x\to\infty} a^x \begin{cases} \to \infty & a>1 \\ \to 1^\infty & a=1 \text{ (belirsiz)} \\ \to 0 & a<1 \end{cases}}$$

Ö $\lim\limits_{x\to\infty} e^{-x}.\ln x = 0.\ln\infty = 0.\infty$

$$\lim_{x\to\infty} \frac{\ln x}{e^x} = \frac{\ln\infty}{e^\infty} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{e^x} = \frac{\frac{1}{\infty}}{e^\infty} = \frac{1}{\infty} = 0$$

Ö
$$\lim_{x\to 0} \frac{e^{x^2}-e^{x}}{x} = \frac{1-1}{0} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{2x.e^{x^2}-e^{x}}{1} = \frac{0e^0-e^0}{1} = -1$$

Ö
$$\lim_{x\to 0} \frac{e^{\sin^2 x}-e^{2x}}{x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{2\sin x.\cos x.e^{\sin^2 x}-2e^{2x}}{1} = \frac{0-2}{1} = -2$$

Ö
$$\lim_{x\to 0} \frac{2^x-3^x}{x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{2^x.\ln 2-3^x.\ln 3}{1} = \frac{\ln 2-\ln 3}{1} = \ln\frac{2}{3}$$

Ö
$$\lim_{x\to 1} \frac{e^{x-1}-2^{x-1}}{x-1} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{e^{x-1}-2^{x-1}.\ln 2}{1}$$

$$y = 2^{x-1}$$
$$\ln y = \ln 2^{x-1}$$
$$\ln y = x-1.\ln 2$$
$$\frac{y'}{y} = \ln 2$$
$$y' = 2^{x-1}.\ln 2$$

$$\lim_{x\to 1} \frac{1-\ln 2}{1} = \ln e - \ln 2 = \ln\frac{e}{2}$$

Ö
$$\lim_{x\to\infty} e^{-x^2}\ln(x^2+1) = 0.\infty$$

$$\lim_{x\to\infty} \frac{\ln(x^2+1)}{e^{x^2}} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{2x}{x^2+1}}{2xe^{x^2}} = \lim_{x\to\infty} \frac{2x}{(x^2+1).2x.e^{x^2}} = 0$$

$$\lim_{x\to 1} (x^2-1)^{\frac{1}{x-1}} = 0^{\frac{1}{0}} = 0^{\infty} = 0$$

$$\lim_{x\to 1} \ln(x^2-1)^{\frac{1}{x-1}} = \lim_{x\to 1} \frac{1}{x-1} \cdot \ln(x^2-1) = \frac{1}{0} \cdot \ln 0 = \infty \cdot (-\infty) = -\infty$$

$$\ln y = -\infty = -\infty \cdot 1 = -\infty \cdot \ln e = \ln e^{-\infty} \Rightarrow y = e^{-\infty} = 0$$

$$\boxed{\begin{array}{c} \ln 0 = a \cdot 1 = a \cdot \ln e = \ln e^{a} \\ 0 = e^{a} \to a = -\infty \\ \ln 0 = -\infty \end{array}}$$

$$\lim_{x\to\infty} (x^2+1)^{\frac{1}{x+1}} = 1$$

$$\lim_{x\to\infty} (x^2+1)^{\frac{1}{x+1}} = \infty^{0}$$

$$\lim_{x\to\infty} \ln(x^2+1)^{\frac{1}{x+1}} = \lim_{x\to\infty} \frac{1}{x+1} \cdot \ln(x^2+1) = \frac{1}{\infty} \cdot \ln\infty \qquad \boxed{0^{\infty} = 0}$$

$$= 0 \cdot \infty$$

$$\lim_{x\to\infty} \frac{\ln(x^2+1)}{x+1} = \frac{\infty}{\infty} \Rightarrow \lim_{x\to\infty} \frac{\frac{2x}{x^2+1}}{1} = 0$$

$$\ln y = 0 = \ln e^{0} = \ln 1 \Rightarrow y = 1$$

* $\frac{0}{0}$ ve $\frac{\infty}{\infty}$ / $\infty - \infty$, $0 \cdot \infty$, 0^0, ∞^0, $1^{\infty} \Rightarrow \frac{0}{0}$ veya $\frac{\infty}{\infty}$'a çevrilir

$$\lim_{x\to\infty} \left(\frac{x^3-1}{x^3+1}\right)^{x^3} \Rightarrow \lim_{x\to\infty} \left(\frac{x^3-1}{x^3+1}\right) = \frac{\infty}{\infty}$$

$$= 1^{\infty} \qquad \lim_{x\to\infty} \frac{3\cancel{x^2}}{3\cancel{x^2}} = 1$$

$$\lim_{x\to\infty} x^3 \ln\left(\frac{x^3-1}{x^3+1}\right) = \infty \cdot \ln 1 = \infty \cdot 0$$

$$\lim_{x\to\infty} \frac{\ln\left(\frac{x^3-1}{x^3+1}\right)}{\frac{1}{x^3}} = \frac{\ln 1}{\frac{1}{\infty}} = \frac{0}{0}$$

$$\lim_{x\to\infty} \frac{\dfrac{\frac{3x^2(x^3+1)-3x^2(x^3-1)}{(x^3+1)^2}}{\frac{x^3-1}{x^3+1}}}{-\frac{3}{x^4}} = \lim_{x\to\infty} \frac{-2x^2(x^3+1) \cdot x^4}{(x^3+1)^2 (x^3-1)} = \underbrace{\frac{-2}{1}}_{\text{katsayı oranı}} = -2$$

DEVAMI ARKADA

21.2

$\ln y = -2$

$\ln y = -2 \ln e = \ln e^{-2}$

$y = e^{-2}$

$$\lim_{x \to \infty} \left(\frac{x^3 - 1}{x^3 + 1} \right)^{x^3} = e^{-2}$$

$$\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{1/x} = \left(\frac{\sin 0}{0} \right)^{1/0} = \left(\frac{0}{0} \right)^{1/0} \Rightarrow \lim_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\cos x}{1} = 1$$

$$\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{1/x} = 1^{\infty}$$

$$\lim_{x \to 0} \ln \left(\frac{\sin x}{x} \right)^{1/x} = \lim_{x \to 0} \frac{1}{x} \cdot \ln \frac{\sin x}{x} = \frac{1}{0} \cdot \ln 1 = \infty \cdot 0$$

$$\lim_{x \to 0} \frac{\ln \left(\frac{\sin x}{x} \right)}{x} = \frac{\ln 1}{0} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\dfrac{\dfrac{x \cdot \cos x - \sin x}{x^2}}{\dfrac{\sin x}{x}}}{1} = \lim_{x \to 0} \frac{x \cos x - \sin x}{\cancel{x^2}} \cdot \frac{x}{\sin x} =$$

$$= \lim_{x \to 0} \frac{(x \cos x - \sin x)}{x \cdot \sin x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\cos x - x \sin x - \cos x}{\sin x + x \cos x} = \lim_{x \to 0} \frac{-x \cdot \sin x}{\sin x + x \cos x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{-\sin x - x \cos x}{\cos x + \cos x - x \sin x} = \frac{0}{2} = 0$$

$\ln y = 0 = 0 \cdot \ln e = \ln e^0$

$y = e^0 = 1$

Ö $\lim_{x\to\infty}\left(\frac{e^x}{x+1}\right)^{\frac{1}{x}} = \left(\frac{\infty}{\infty}\right)^0 \Rightarrow \lim_{x\to\infty}\frac{e^x}{x+1} = \frac{\infty}{\infty}$

$= \infty^{\frac{1}{\infty}} = \infty^0$ $\quad \lim_{x\to\infty}\frac{e^x}{1} = \infty$

$\lim_{x\to\infty}\frac{1}{x}\ln\left(\frac{e^x}{x+1}\right) = \frac{1}{\infty}\ln\infty = 0.\infty$

$\lim_{x\to\infty}\frac{\ln\left(\frac{e^x}{x+1}\right)}{x} = \frac{\infty}{\infty}$

$$\lim_{x\to\infty}\frac{\dfrac{\dfrac{e^x(x+1)-e^x}{(x+1)^2}}{\dfrac{e^x}{x+1}}}{1} = \frac{xe^x}{(x+1)^2}\cdot\frac{(x+1)}{e^x} = \frac{x}{x+1}$$

$\lim_{x\to\infty}\underbrace{\frac{x}{x+1}}_{\text{katsayı oranı}} = 1$

$\ln y = 1 \Rightarrow y = e$

$\lim_{x\to\infty}\left(\frac{e^x}{x+1}\right)^{\frac{1}{x}} = e$

Ö $\lim_{x\to 0}\left[\left(\frac{\sin x}{x}\right)^{\frac{1}{x}}\right]^{\cos x} = (1^\infty)^{\cos x} \Rightarrow \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}} = 1$

$\lim_{x\to 0}(1)^{\cos x} = 1^1 = 1$

Ö $\lim_{x\to\infty}\left[\left(\frac{e^x}{x+1}\right)^{1/x}\right]^{e^x} = [e]^{e^x} = \infty$

Ö $\lim_{x\to 0}\left[\left(\frac{\sin x}{x}\right)^{\frac{1}{x}}\right]^{\frac{1}{e^x-1}} = 1^\infty$

$\lim_{x\to 0}\frac{1}{e^x-1}\ln\left(\frac{\sin x}{x}\right)^{\frac{1}{x}} = \frac{1}{0}\cdot\ln 1 = \infty\cdot 0$

⋮

—o— Mini Sınav Soruları Ve Çözümleri —o—

1- $x = \ln(t+\sqrt{1+t^2})$, $y = \frac{1}{\sqrt{1+t^2}}$ ise $\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\frac{1}{2}(1+t^2)^{-3/2}\cdot 2t}{\frac{1+\frac{1}{2}(1+t^2)^{-1/2}\cdot 2t}{t+\sqrt{1+t^2}}} = -\frac{1}{2}$$

2- $f(x) = \sin x$

$\sin x = x - \frac{x^3}{3!} + \bar{R}_5(x)$ $\quad \bar{R}_5(\theta x) = \frac{f^5(\theta x)}{5!}x^5$

$x = \frac{\pi}{10}$

$\sin 18° = \frac{\pi}{10} - \frac{\left(\frac{\pi}{10}\right)^3}{3!} = 0{,}309$

3- $y = (1+\operatorname{arctg}^2 x)^{\sin^2 x}$ $\quad x=0$ için $\frac{dy}{dx} = ?$

$\ln y = \sin^2 x \ln(1+\operatorname{arctg}^2 x)$

$\frac{y'}{y} = 2\sin x\cos x\cdot\ln(1+\operatorname{arctg}^2 x) + \sin^2 x\,\frac{2\operatorname{arctg}x\cdot\frac{1}{1+x^2}}{1+\operatorname{arctg}^2 x}$

x=0 için $y' = 0$

4- $y=\left(2+e^{\sin 2x}\right)^{x^2+1}$ $x=0$ için $\frac{dy}{dx}=?$

$\ln y = x^2+1 . \ln\left(2+e^{\sin 2x}\right)$

$\frac{y'}{y} = 2x . \ln\left(2+e^{\sin 2x}\right) + (x^2+1) . \frac{2\cos 2x . e^{\sin 2x}}{2+e^{\sin 2x}}$

$x=0 \Rightarrow y' = \frac{2}{3} . 3 = 2$

5- $y^2 = e^{xy}$ $x=0$ için T.D ve N.D = ?

$y^2 = e^0 = 1 \Rightarrow y = \mp 1$

$2\ln y = xy$

$2\frac{y'}{y} = y + xy'$

* $y' = \frac{1}{2} = m \Rightarrow x=0 , y=1$

$y - y_0 = m(x - x_0)$

$y - 1 = \frac{1}{2}(x-0)$

$y - 1 = \frac{1}{2}x$

$\boxed{y = \frac{x}{2}+1} \to$ T.D

$m . m' = -1 \Rightarrow m' = -2$

$y - 1 = -2x$

$\boxed{y = -2x+1} \to$ N.D

* $x=0 , y=-1$ için $y' = \frac{1}{2}$

$y-(-1) = \frac{1}{2}(x-0)$

$y+1 = \frac{1}{2}x$

$\boxed{y = \frac{1}{2}x - 1} \to$ T.D

$y+1 = -2x$

$\boxed{y = -2x-1} \to$ N.D

6- $\lim_{x\to 0} \frac{\sin x^2 - \ln(1+x^2)}{1-\cos x^2} = \frac{0}{0}$

$\lim_{x\to 0} \frac{2x.\cos x^2 - \frac{2x}{1+x^2}}{2x\sin x^2} = \lim_{x\to 0} \frac{\cos x^2 - \frac{1}{1+x^2}}{\sin x^2} = \frac{0}{0}$

$\lim_{x\to 0} \frac{-\cancel{2x}\sin x^2 + \frac{\cancel{2x}}{(1+x^2)^2}}{\cancel{2x}\cos x^2} = \lim_{x\to 0} \frac{-\sin x^2 + \frac{1}{(1+x^2)^2}}{\cos x^2} = 1$

7- $\lim_{x\to 0} \left(\frac{1}{x}\right)^{\ln\cos x} = \infty^{0}$

$y = \left(\frac{1}{x}\right)^{\ln\cos x}$

$\ln y = \ln\cos x \cdot \ln\frac{1}{x}$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \ln\cos x \cdot \ln\left(\frac{1}{x}\right) \quad (0 \cdot \infty)$

$= \lim_{x\to 0} \frac{\ln\cos x}{\frac{1}{\ln\left(\frac{1}{x}\right)}} = \frac{\ln\cos 0}{\frac{1}{\left(\ln\frac{1}{0}\right)}_{\infty}} = \frac{0}{0}$

$\lim_{x\to 0} \frac{-\frac{\sin x}{\cos x}}{\frac{\frac{1}{x}}{-\frac{1}{x^2}}} = \lim_{x\to 0} \frac{\tan x}{x} = \frac{0}{0}$

$\lim_{x\to 0} \frac{1/\cos^2 x}{1} = \underline{\underline{1}}$

$\ln y = 1 \cdot \ln e \quad 1 \cdot 1 = 1 \cdot \ln e$

$y = e^1$

$\lim_{x\to 0} y = e$

ANALİZ II VİZE SORULARI

① $x = e^{-2t}\cos 3t$

$y = e^{-2t}\sin 3t$

i) $t=4$ için $\vartheta^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = ?$

$\frac{dx}{dt} = -2e^{-2t}\cdot\cos 3t - 3\cdot e^{-2t}\sin 3t = -2e^{-8}\cos 12 - 3e^{-8}\sin 12 \cong -8{,}5\times 10^{-4}$

$\frac{dy}{dt} = 2e^{-2t}\sin 3t + 3e^{-2t}\cos 3t \cong -2e^{-8}\sin 12 + 3e^{-8}\cos 12 \cong 8{,}4\times 10^{-4}$

ii) $\frac{dy}{dx} = \frac{dy/dt}{\frac{dx}{dt}} \cong -0{,}98$

② i) $z^4 = 1+i$

$z = x+iy \Rightarrow x=1 \quad y=1 \Rightarrow r=\sqrt{2}$

$x = r\cos\theta \longrightarrow 1 = \sqrt{2}\cos\theta \longrightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$y = r\sin\theta \longrightarrow 1 = \sqrt{2}\sin\theta \longrightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}$

$\theta = 2\pi k + \frac{\pi}{4}$

$z = x+iy = r(\cos\theta + i\sin\theta)$

$z^4 = \sqrt{2}\left[\cos\left(2\pi k + \frac{\pi}{4}\right) + i\sin\left(2\pi k + \frac{\pi}{4}\right)\right]$

$z_k = (\sqrt{2})^{\frac{1}{4}}\left[\cos\left(2\pi k + \frac{\pi}{4}\right) + i\sin\left(2\pi k + \frac{\pi}{4}\right)\right]^{1/4}$

$z_0 = 2^{\frac{1}{8}}\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right)$

$z_1 = 2^{1/8}\left(\cos\frac{9\pi}{16} + i\sin\frac{9\pi}{16}\right)$

$z_2 = 2^{1/8}\left(\cos\frac{17\pi}{16} + i\sin\frac{17\pi}{16}\right)$

$z_3 = 2^{1/8}\left(\cos\frac{25\pi}{16} + i\sin\frac{25\pi}{6}\right)$

ii) $\sin 12^\circ = ?$ $\qquad x = \frac{\pi}{15}$

$\sin x = x - \frac{x^3}{3!} + R_5$

$\sin\frac{\pi}{15} = \frac{\pi}{15} - \frac{(\pi/15)^3}{3!} \cong 0{,}207$

③

$y=(1+e^{arctg2x^2})^{1+x^4}$

$x=0 \rightarrow y=(1+e^0)^{1+0}=2$

$\ln y = 1+x^4 . \ln 1+e^{arctg 2x^2}$

$\frac{y'}{y} = 4x^3 . \ln(1+e^{arctg2x^2}) + (1+x^4) . \frac{\frac{4x}{1+(2x^2)^2} e^{arctg 2x^2}}{(1+e^{arctg2x^2})}$

$x=y'=0$

$y'' = 12x^2 . \ln(1+e^{arctg 2x^2}) . y + 4x^3 . \frac{4x}{1+(2x^2)^2} . e^{arctg 2x^2} / 1+e^{arctg2x^2} . y +$

$4x^3 \left(\frac{4xe^{arctg 2x^2}}{\underbrace{(1+4x^4)(1+e^{arctg2x^2})}_{payda}} \right) y + (1+x^4) . \left[\frac{4e^{arctg 2x^2} + 4x . \frac{4x}{1+4x^4} . e^{arctg2x^2}}{(1+4x^4)^2} \right.$

$\frac{. (1+4x^4)(1+e^{arctg 2x^2}) - 4xe^{arctg 2x^2} (16x^3(1+e^{arctg 2x^2}) + 1+4x^4 . \frac{4x}{1+4x^4} e^{arctg 2x^2}}{(1+e^{arctg2x^2})^2}$

$. y =$

$y'' = 4$

④ i) $\ln y^2 = e^{xy}$ $\quad x=0 \quad$ T.D, N.D

$2 \ln y = e^{xy} \longrightarrow 2 \frac{y'}{y} = (y + xy') . e^{xy}$

$2 \ln y = 1$ $\quad 2 \frac{y'}{e^{1/2}} = (e^{1/2} + 0) . e^0 =$

$\ln y = \frac{1}{2} \quad y = e^{\frac{1}{2}}$

$2y' = e^{\frac{1}{2}} . e^{\frac{1}{2}} \Rightarrow y' = \frac{e}{2}$

T.D $\Rightarrow y - e^{\frac{1}{2}} = \frac{e}{2} x$

$m.m' = -1$ $\quad y - e^{1/2} = -\frac{2}{e} x \Rightarrow$ N.D

$m' = -\frac{2}{e}$

ii) $y = e^{x^3} \longrightarrow \frac{dy}{dx} = 3x^2 \cdot e^{x^3}$

$y + dy = e^{(x+dx)^3} = e^{x^3 + 3x^2dx + 3xdx^2 + dx^3}$ $\quad dx << 1$

$= e^{x^3} \cdot e^{3x^2dx}$ $\quad dx^2 \simeq 0$

$e^{3x^2dx} = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n\,3x^2dx} = 1 + \frac{1}{\cancel{n}} \cdot 3x^2dx\cancel{n} + \frac{1}{n^2}\frac{3x^2dx(3x^2dx - 1)}{2!} \cdots]$

$e^{3x^2dx} \simeq 1 + 3x^2dx$

$y + dy = e^{x^3}(1 + 3x^2dx)$

$$\lim_{dx\to 0} \frac{dy}{dx} = \frac{(y+dy) - y}{dx} = \frac{e^{x^3}(1 + 3x^2dx) - e^{x^3}}{dx} = \frac{3x^2e^{x^3}\cancel{dx}}{\cancel{dx}}$$

$\frac{dy}{dx} = 3x^2 \cdot e^{x^3}$

(5) i) $\lim_{x\to 2} \left[\frac{e^{x^2-4} - 3^{x-2}}{x-2}\right]^{x-2}$

$\lim_{x\to 2} (x-2)\ln\left(\frac{e^{x^2-4} - 3^{x-2}}{x-2}\right) = \lim_{x\to 2} (x-2) \cdot \lim_{x\to 2} \left(\frac{e^{x^2-4} - 3^{x-2}}{x-2}\right)$

$= 0 \cdot \lim_{x\to 2} \frac{1-1}{0} = \frac{0}{0}$

$\Rightarrow 0 \cdot \lim_{x\to 2} \ln\left(\underbrace{\frac{2x \cdot e^{x^2-4} - \ln 3 \cdot 3^{x-2}}{1}}_{4-\ln 3}\right) = 0$

$\lim_{x\to 2} y = e^0 = 1$ //

ii) $\lim_{x\to 0} \left(\frac{\cos x}{\cosh x}\right)^{\frac{1}{x^2}} = 1^\infty$

$$\lim_{x\to 0} \frac{\ln \frac{\cos x}{\cosh x}}{x^2} = \frac{0}{0} = \lim_{x\to 0} \frac{\frac{\frac{-\sin x \cdot \cosh x - \cos x \cdot \sinh x}{(\cosh x)^2}}{\cos x / \cosh x}}{2x} =$$

$\lim_{x\to 0} \frac{-tg\,x - tgh\,x}{2x} = \frac{0}{0}$

$\lim_{x\to 0} \frac{-1/\cos^2 x - 1/\cosh^2 x}{2} = \frac{-2}{2} = -1 \Rightarrow \lim_{x\to 0} y = e^{-1}$