Evrenin başında ne vardı? Sonunda ne olacak?

Metin Arık

Prof. Dr. K. Gediz Akdeniz'in 70. yaşı onuruna

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GEDİZ'İN İZİNDE

Türk Fizik Derneği Genel Başkanlığı (1987-1997)
Balkan Fizik Birliği Başkanlığı (1989-1997)

K.G. Akdeniz, M. Arik and E. Rizaoglu, "Coasting Big-Bang", Phys. Lett. B 321, 329 (1994)

K.G. Akdeniz, M. Arik, H. Mutus and E. Rizaoglu,
"Coasting Kaluza-Klein cosmology", Mod. Phys. Lett. A 6, 1543 (1991)

M. Arik,
"The analogue potential of Kaluza-Klein cosmology with Euler-form actions",
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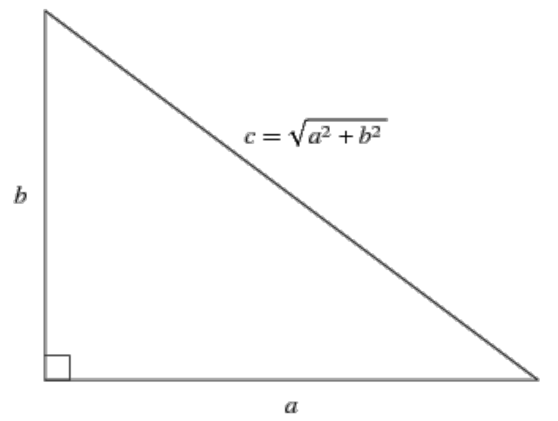

Pisagor teoremi: $c^2=a^2+b^2$ MÖ 500
Düz uzay metriği: $ds^2=dx^2+dy^2+dz^2$ Descartes 1600
Zaman-Uzay metriği: $ds^2=dt^2 - a^2(t)$(maksimal simetrik uzay metriği)
Friedmann-Lemaitre-Robertson-Walker 1920-1950

maksimal simetrik uzay= R^3, S^3, H^3

$ds^2=dt^2 - a^2(t)$ (S^3 metriği)

$ds^2=dt^2 -$ (başlangıçta t^2 sonra t) (S^3 metriği) Tatlı patlama

$ds^2=dt^2 - t(T-t)$ (S^3 metriği) Büyük patlame

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PHYSICS LETTERS B

Coasting big bang ☆

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We consider a cosmological model in which the early universe is dominated by the equation of state $p=-\rho/3$ corresponding to a coasting cosmology. This equation of state was first obtained in the context of cosmic string theories whereas its one-parameter generalization describing a coasting early universe followed by a radiation-dominated universe was first derived from Kaluza–Klein theories based on Euler form actions. We show that in such a model the temperature is almost absolute zero at the beginning and the universe quickly heats up to maximum temperature.

The past few decades have been remarkable from the point of relativistic cosmology. On the one hand the discovery of the cosmic microwave background radiation has confirmed that the universe at one time was very hot as predicted by the hot big-bang model. On the other hand it has turned out that this standard model of the universe has some problems which can only be solved by introducing new ideas. For example in the standard Robertson–Walker cosmology it is possible to solve the horizon problem only with an equation of state with negative pressure. It turns out that if in the early universe $p\leqslant -\rho/3$ then there is no horizon problem [1,2]. The case $p=-\rho/3$ corresponds to the coasting universe [3] which expands linearly in time. Such a behaviour, according to Vilenkin [4], is obtained by a universe dominated by cosmic strings. A similar equation of state has also been derived by Gasperini et al. [5] for unstable strings, i.e., non-oscillating string configurations whose proper amplitude tends to evolve, asymptotically, like the radius of the universe.

Various attempts have been made to initiate the universe with a string phase. Besides solving the problems of standard cosmology, these models also try to provide an explanation as to why the space dimension of the universe is three. In these works, the strings considered are either supersymmetric or classical strings obeying the Nambu–Goto equation. These studies have once more shown the importance of the statistical and thermodynamical properties of a string gas. The results of these studies depend on the single string spectrum specified by the particular theory.

Matter which behaves differently from point particles can cause a difference in the dynamical behaviour of the universe. Here we will describe the global aspects of the dynamics of such a universe by investigating a Friedmann–Robertson–Walker cosmology with an unusual and interesting equation of state which has first been derived from Kaluza–Klein considerations [6–8]

$$p=\frac{\rho}{3}\frac{1-\rho/\rho_0}{1+\rho/\rho_0}\,, \qquad (1)$$

where ρ_0 is a constant. Covariant conservation of the energy–momentum tensor leads to the continuity equation

$$\dot{\rho}+3(\rho+p)\,\frac{\dot{R}}{R}=0\,. \qquad (2)$$

This equation together with the equation of state given by (1) yields

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$$\rho=\rho_0\left[\sqrt{1+\left(\frac{R_1}{R}\right)^4}-1\right], \tag{3}$$

where R_1 is the integration constant. As can be seen from (3) the energy density ρ is a monotonically decreasing function of the world-radius R. As the world-radius tends to zero, the energy density tends to infinity. Hence in the early universe the equation of state given by (1) reduces to $p=-\rho/3$. With this equation of state the Friedmann equations give $R\sim t$ and $\rho\sim R^{-2}$ and zero deceleration. These are the properties of a cosmic-string-dominated universe [3,4]. Because of these properties we call the very early stage of the universe the cosmic string stage. In the late stages, as the world-radius becomes very large, the energy density tends to zero and $p=\rho/3$, corresponding to a radiation-dominated universe. To find the string content and radiation content of the universe we use

$$\rho=\rho_r+\rho_s, \quad p=p_r+p_s,$$

$$p_r=\rho_r/3, \quad p_s=-\rho_s/3,$$

together with (1) to obtain

$$\rho_r=\rho_0\frac{\rho/\rho_0}{1+\rho/\rho_0}, \quad \rho_s=\rho_0\frac{(\rho/\rho_0)^2}{1+\rho/\rho_0}. \tag{4}$$

These equations show that in the beginning as $\rho\to\infty$, there is some radiation contributing an amount ρ_0 to the energy density. The radiation density starts from this value and continually decreases.

For a string-dominated universe the energy density is a function of temperature only, provided that there are no winding modes. In the framework of Kaluza–Klein theory there are no winding modes, provided that the space, including internal space, is simply connected. Hence we consider an energy density which only depends on temperature and investigate the consequences of the equation of state (1)

According to the second law of thermodynamics, the entropy of the universe is a function $S(T,V)$ with

$$\mathrm{d}S(T,V)=\frac{1}{T}\{\mathrm{d}[\rho(T)V]+p(T)\mathrm{d}V\}. \tag{5}$$

So the equality of the mixed derivatives yields

$$\frac{\mathrm{d}p(T)}{\mathrm{d}T}=\frac{1}{T}[\rho(T)+p(T)]. \tag{6}$$

In our case, the use of (1) and (3) results in

$$\frac{\mathrm{d}\rho}{\mathrm{d}T}=\frac{2}{T}\frac{\rho(1+\rho/\rho_0)(2+\rho/\rho_0)}{1-2\rho/\rho_0-(\rho/\rho_0)^2},$$

which by integration gives

$$T=\sqrt{2}T_1\frac{[\rho/\rho_0(2+\rho/\rho_0)]^{1/4}}{1+\rho/\rho_0}, \tag{7}$$

where T_1 is the integration constant. By use of (3) in (7) we obtain

$$T=\sqrt{2}T_1\frac{R_1R}{\sqrt{R_1^4+R^4}}. \tag{8}$$

From (8) we see that the temperature starts from zero at $R=0$ and increases to its maximum value T_1 at $R=R_1$. This is the most striking feature of our model. Nevertheless this is not surprising since it is well known that the temperature of a non-interacting gas of continuously extended objects cannot exceed a maximum temperature, known as the Hagedorn temperature [9]. The new result is the steep increase of temperature from almost absolute zero to T_1. A detailed investigation of other thermodynamical quantities shows that there is indeed a phase transition at $T=T_1$.

We would like to mention that in string theories containing winding modes the energy density depends on both the temperature and the volume. In this case the temperature initially remains constant. For example for a universe filled with type II superstrings such a T–R curve was obtained by Brandenberger and Vafa by using thermodynamical arguments without any reference to the dynamics of gravity [10].

By use of (6) in the energy conservation equation, (2), we get [2]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{R^3}{T}[\rho(T)+p(T)]\right)=0. \tag{9}$$

On the other hand using (6) in (5) one obtains

$$\mathrm{d}S(V,T)=\frac{1}{T}\mathrm{d}\{[\rho(T)+p(T)]V\}$$

$$-\frac{V}{T^2}[\rho(T)+p(T)]\mathrm{d}T,$$

so, except for a plausible additive constant,

$$S(V,T)=\frac{V}{T}[\rho(T)+p(T)]\,. \tag{10}$$

When we consider (9) and (10) together we see that as a result of energy conservation the universe evolves with constant entropy. So the form of our T–R curve is implicitly dictated by adiabaticity.

In order to calculate the (constant) value of the entropy we use (1) and (10) to get

$$S=\frac{2\pi^2R^3}{T}\frac{2}{3}\rho\frac{2+\rho/\rho_0}{1+\rho/\rho_0}\,.$$

Substituting for T from (8) and recalling (3) this gives

$$S=\frac{2(2\pi^2)\rho_0R_1^3}{3T_1}\,. \tag{11}$$

In the above formulae we have denoted the quantities corresponding to the maximum temperature with the subindex 1 and the ones corresponding to zero pressure with the subindex 0. Both conditions approximately correspond to the instant when the universe passes from the cosmic-string-dominated stage to the radiation-dominated one. Using the above formulae one can easily compute

$$R_0=0.76\,R_1\,,\quad T_0=0.93\,T_1\,,\quad \rho_0=2.4\,\rho_1\,. \tag{12}$$

Determining a value of the parameter R_1 will be one of the main objectives of this paper. (11) shows that if ρ_0 and T_1 are chosen to be the Planck density and the Planck temperature the large value of the entropy of the observed universe is a direct consequence of the large value of the parameter R_1 compared to the Planck length.

Using the present entropy of the universe which in dimensionless units is approximately 10^{87} one obtains that R_1 is approximately 10^{29} Planck lengths which is roughly the geometric mean of the present size of the universe and the Planck length. Although this choice for the parameter R_1 seems quite natural, the fact that it turns out to be much larger than the Planck length is the flatness problem which in the context of the present model is not solved.

Thus we find that for densities greater than the Planck density, the universe behaves as a cosmic-string-dominated one in the framework of homogeneous and isotropic (3+1)-dimensional general relativity. We know that at the Planck density the quantum gravitational effects dominate the space–time structure. Hence in Kaluza–Klein models with Euler form actions the quantum gravitational effects manifest themselves in the effective (3+1)-dimensional relativistic cosmology as a universe evolving spontaneously from the cosmic-string-dominated stage into a radiation-dominated one.

Comparing our model with that of ref. [10] we see that although superstrings and cosmic strings in today's universe are entirely different objects, in the early universe when the universe is small their effects on cosmology can be identical. One interesting observation in this regard is that the temperature (8) derived in our model is invariant under the inversion $R/R_1\to R_1/R$. Such a behaviour is precisely what is expected in superstring theory. However our value of $R_1\approx 10^{29}$ in Planck units is quite different from the value $R_1=1$ advocated in ref. [10].

Now, we turn to discuss the well-known horizon problem of standard cosmology which forces us to abandon the view that radiation dominance extends until the very beginning of the universe. The condition for the solution of the horizon problem is $d_{\rm dec}\sim d_{\rm H}(t_{\rm dec})$, where $d_{\rm dec}$ is the proper size of the part of the universe that is now visible taken at the time of decoupling, and $d_{\rm H}(t_{\rm dec})$ is the maximum distance which can be covered before $t_{\rm dec}$ by a signal travelling with the speed of light, i.e., the proper distance of the particle horizon at decoupling [11]. We know that $d_{\rm dec}=d(t_{\rm dec})=6.9\times10^{24}$ cm and $t_{\rm dec}=3\times10^{12}$ s [11]. Let us calculate

$$d_{\rm H}(t)=cR(t)\int\limits_{R_{\rm i}}^{R(t)}\frac{{\rm d}R}{R\dot R} \tag{13}$$

for our model. Substituting the value of $\dot R$ from the first Friedmann equation (for simplicity $k=0$) and using (3) we get

$$d_{\rm H}(t)=-\frac{c}{2}\left(\frac{3}{8\pi G\rho_0}\right)^{1/2}r(t)\times\int\limits_{V_{\rm in}}^{V(t)}\frac{1+v^2}{1-v^2}\frac{1}{v^{3/2}}\,{\rm d}v \tag{14}$$

where $r(t)=R(t)/R_1$, $v=\sqrt{r^4+1}-r^2$ and $R_{\rm in}$ is the initial world-radius of the order of a few Planck lengths. Evaluating the integral one finds

$$d_H(t_{dec}) = \frac{c}{R_1}\left(\frac{3}{8\pi G\rho_0}\right)^{1/2}$$

$$\times\Bigg(v_{dec}^{-1/2} - v_{in}^{-1/2}$$

$$+\tfrac{1}{2}\log\left|\frac{\sqrt{v_{dec}}-1}{\sqrt{v_{dec}}+1}\right| - \tfrac{1}{2}\log\left|\frac{\sqrt{v_{in}}-1}{\sqrt{v_{in}}+1}\right|$$

$$+ \text{Arctg}\sqrt{v_{dec}} - \text{Arctg}\sqrt{v_{in}}\Bigg) R(t_{dec}) . \qquad (15)$$

Using the accepted value of $R(t_{dec})$ and approximating $\sqrt{v_{in}}$,

$$\sqrt{v_{dec}} \cong \frac{1}{\sqrt{2}} \times 10^{-29}, \quad \sqrt{v_{in}} \cong 1 - \tfrac{1}{2} r_{in}^2 ,$$

we get

$$d_H(t_{dec}) = \frac{3}{\sqrt{28}} R(t_{dec})$$

and

$$\frac{d_H(t_{dec})}{d_{dec}} = \frac{3}{\sqrt{28}} \frac{R(t_{dec})}{d_{dec}}$$

$$= \frac{3}{\sqrt{28}} \frac{1}{6.9} \times 10^{-24} R(t_{dec}) . \qquad (16)$$

Today it is known that decoupling occurs just after equilibrium, at the beginning of the matter era. Now we make an approximation and continue to use the same analytic formula for t [8] even after equilibrium:

$$t = t_{s.c.} - \left(\frac{3}{64\pi G\rho_0}\right)^{1/2} \text{Arctg}\sqrt{\frac{\rho}{2\rho_0}} ,$$

where $t_{s.c.}$ is the expression for the time in the standard radiation-dominated cosmology. The second term on the right-hand side has an upper bound of 10^{-44} s and is totally negligible at the time of decoupling. Hence

$$t_{dec} = (t_{s.c.})_{dec} = \left(\frac{3}{32\pi G\rho_{dec}}\right)^{1/2} .$$

This gives

$$\rho_{dec} = 10^{19} \text{g cm}^{-3} .$$

Solving R from (3) gives

$$R_{dec} \cong 10^{24} \text{cm} . \qquad (17)$$

Combining (16) and (17) we get

$$d_H(t_{dec}) \sim d_{dec} ,$$

and this shows that at the time of decoupling the comoving volume which contains our presently observed homogeneous universe was causally connected.

In conclusion we would like to emphasize that once eq. (1) is assumed the only other assumption needed to fix the model is the size of the parameter R_1. Eq. (1) can be derived [8] within the context of the general Kaluza–Klein cosmology [7] with constant internal space size and zero internal pressure. The constancy of the radius of internal space guarantees that the gauge coupling remains constant as the universe evolves, as well as the fact that the continuity equation is exactly the same as for a (3+1)-dimensional cosmology. The condition of zero internal pressure follows from the fact that the energy–momentum tensor is purely (3+1) dimensional.

Our choice of the value $R_1 = 10^{29}$ Planck lengths is derived by fitting the total conserved entropy of radiation only. At this stage our model completely ignores the matter-dominated late stage of the universe. However, it will correctly reproduce the present 2.7 K microwave background radiation temperature.

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